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Threshold cointegration and threshold dynamics

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Threshold cointegration and threshold dynamics

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TABLE OF CONTENTS

ABSTRACT	iv
CHAPTER 1. INTRODUCTION	1
CHAPTER 2. MOTIVATION AND LITERATURE REVIEW	9
CHAPTER 3. METHODOLOGIES	23
CHAPTER 4. DATA	45
CHAPTER 5. ESTIMATION RESULTS	62
CHAPTER 6. FORECASTING PERFORMANCE EVALUATION	116
CHAPTER 7. CONCLUSIONS	164
APPENDIX A: ESTIMATION OF THE LO-ZIVOT MODEL	167
APPENDIX B: ESTIMATION OF THE HANSEN-SEO MODEL	170
APPENDIX C: SOME MEASURES OF FORECAST ACCURACY	174
REFERENCES	176
ACKNOWLEDGMENTS	182

ABSTRACT

This study utilized monthly averages of daily rates for the 10-year constant maturity Treasury note, the Ibbotson Bond Index with maturity of 20-year Treasury Index, and Moody's Aaa and Baa seasoned bond indices to investigate the threshold behavior of interest rates pairs. The data covered the period from January 1960 to December 1997, with a total of 456 observations for each variable. Three (Lo-Zivot 2001, Hansen-Seo 2002, and Enders-Siklos 2001) different non-linear, discontinuous, asymmetric time-series econometric alternatives were applied to investigate the dynamics of the four interest rates pairs. Forecasting accuracy evaluation was utilized for model evaluation by applying one-step-ahead up to six-step-ahead forecasts.

Among the findings, it was ascertained that interest spreads are stationary, yet the speeds of adjustment are asymmetric. In a bivariate setting, all of the interest rates pairs followed the threshold cointegration behavior. All the interest rates pairs were shown to be threshold cointegrated. In general, the adjustment speeds were asymmetric and, especially, the threshold estimates were asymmetric in a three-regime environment.

Long run equilibrium relationships existed between Moody's corporate bond indices and Treasury note and Ibbotson bond index. In general, for a one percent increase in Treasury rates (either Treasury note or Ibbotson index), in the long run, it will generate a more than one percent increase in corporate bond indices (Aaa or Baa). Furthermore, the Baa bond index was shown to have a greater sensitivity to interest rate changes than the Aaa bond index.

For the model evaluation side, one-step-ahead forecast to six-step-ahead forecast performance evaluations were conducted for the threshold cointegration models and the counterpart of the linear cointegration models. The results showed that no one particular threshold cointegration model dictated the overall forecasting accuracy. For different interest rates pairs under consideration, different threshold cointegration models offered a better fit. Moreover, all of the linear cointegration models performed relatively less accurate than the threshold cointegration models, which reinforce the empirical applications of the threshold cointegration models.

CHAPTER 1. INTRODUCTION

The credit spread of a given corporate bond is defined as its yield premium minus the yield of a government bond with identical time to maturity. In general, credit spreads exist because of several risks—default, liquidity, operational, legal, systematic, and all other risks inherent in corporate bonds.

Interest in credit spreads stems from a few factors. According to Kao (2000), since 1990, the financial industry has experienced several credit crunches, including the high-yield bond debacle (Junk bonds), large derivative losses (LTCM, 1998), and a global credit/liquidity crisis (the latest Argentina situation, 1999 to 2001). In recent years, academicians and practitioners have shown greater interest in credit-risk pricing models. Fewer than five studies were published per year from 1959 to 1992, whereas from 1993 to 1999, more than 10 studies were published per year; however, more than 30 studies were published in the credit-risk pricing area in 1998 and 1999. One important contributing factor in the past two decades, has been that derivative products have become a desirable part of investment strategies. In particular, credit derivatives with payoffs linked to credit events (e.g., default, change in rating status, credit spreads) have become popular among institutional investors and financial entities.

Several researchers, including Wilson and Jones (1990), Chang and Huang (1990), and Adrangi and Ghazanfari (1996/1997) have documented the seasonality of corporate bonds, e.g., the January effect and the weekday effect, to defy the market efficiency hypothesis. More recently, Collin-Dufresne, Goldstein, and Martin (1999) have shown that hedge funds are extremely sensitive to changes in the credit spread. They examined the determinants of credit spread changes for a sample of individual bonds. Their results show a common factor explains the majority of the variation while a number of variables, which theoretically should explain credit spread changes, have little explanatory power. This finding provides evidence that the gain from studying individual bonds (instead of bond indices) is limited. Pedrosa and Roll (1998) asserted that credit spread risks are non-diversifiable. Hence, their work intensifies the importance of investigating credit spreads. They also found that credit spreads of various indices are affected by some common underlying factors.

It is essential to understand how the pricing of corporate debt is calculated to acquire a deeper understanding of credit risk. The first approach utilizes the *segmentation model*, which divides the pricing factors into three categories: interest rate risk, default risk, and liquidity risk. Fisher (1959), Silvers (1973), and Boardman and McEnally (1981) advocated this approach. The second line of research focuses on the pricing of corporate debt from a long-term perspective, the *market yield premium model*, which examines the effect of risk factors on bonds grouped by risk categories. Advanced by Fons (1987), and Altman and Bencivenga (1995), the market yield premium model analyzes the market yield premium (the average yield spread between risky debt securities and risk-free securities) for holding a risky bond over a long period. This approach shows a positive risk-adjusted return relationship: as the risk of the debt increases, the market yields premium increases. The market yield premium model often applies to the study of the default risk of holding non-investment grade debt.

The third approach focuses on the short-run dynamics of the debt, the *yield spread model*, which includes liquidity risk measures and broadly defined default risk. Pioneered by

Fridson and Jonsson (1995), and Garman and Fridson (1996), the yield-spread model was formulated by defining the dependent variable as the yield on risky debt minus the risk-free rate, and includes both default risk and liquidity risk measures in the analysis as explanatory variables.

The fourth approach is a combination of the yield premium and yield spread models. Barnhill, Joutz, and Maxwell (2000) formulated their model as follows. First, the long-run relationship is estimated through the cointegration techniques of Engle and Granger (1987) and Johansen (1988, 1991). Second, the yield spread part is studied by incorporating a Merrill Lynch aggregate index of the non-investment grade market to assess the default risk and liquidity risk. Third, short-run dynamics and long-run relationships are estimated simultaneously to determine the yields for holding non-investment grade bonds. The researchers concluded that the traditional yield spread model is inadequate and a long-run relationship exists between non-investment grade yields, Treasuries, and default rates.

Neal, Rolph, and Morris (2000), hereafter NRM, also applied Johansen's (1988, 1991) full information maximum likelihood cointegration approach to model the time-series behavior of U.S. government and corporate bond rates. NRM found that government rates are cointegrated with corporate rates and that the time horizon dictates the dynamic relationships between credit spreads and Treasury rates. Intuitively, Treasury and corporate rates have very close co-movements. The cointegrating relation implies that they cannot deviate from each other too much for too long. As mentioned by NRM, this close linkage is not captured in the reduced form bond pricing approach of Duffee (1999) and Duffie and Singleton (1999), or in the structural models such as Merton (1974). Recent development in credit spread option models such as Das and Tufano (1996), and Longstaff and Schwartz (1995b) also failed to capture this linkage.

NRM found that, in the short-run (less than three years), an increase in the Treasury rate would cause the credit spread to narrow. However, for longer time periods (greater or equal to three years), higher Treasury rates will cause credit spreads to widen.[1] Such asymmetric results are not captured by the existing models. This finding has great implications in the areas of modeling bond pricing and credit derivatives. Moreover, NRM's long run results are consistent with models such as Lesseig and Stock (1998), and Duan, Moreau, and Sealey (1995), that enable higher rates to increase credit spreads.

NRM's cointegration approach also provides insights in the area of investment management. They show that corporate bonds are more sensitive to interest rate movements than comparable Treasury rates, since higher Treasury rates widen corporate spreads in the long run. The asymmetric dynamic effects between the short-run and the long-run also offer an insight of the correlational relationship between credit spreads and interest rates that may be *time varying* (e.g., three years is the threshold), a fact also noted by Hund (1999). Current literature, such as Das and Tufano (1996), and Jarrow, Lando and Turnbull (1997), assumes a constant correlation between credit spreads and the risk free rate. NRM claims that "*if the correlation varies over time, it is not clear how to parameterize these models.*" Fortunately, the cointegration technique has the power to overcome this shortcoming.

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[1] Longstaff and Schwartz (1995a) predicted that an increase in the Treasury rate would cause the credit spread to narrow. The argument comes from the relationship between risk-free rate and drift process for firm value. In their model, higher interest rates increase the drift process for firm value and, all else constant, enabling the firm to move further away from a predetermined default barrier. As the default probability is reduced, the credit spread falls. On the other hand, Bernanke and Gertler (1989) implied that higher interest rates, all else constant, would increase credit spreads. In their model, higher rates increase agency problems for borrowers. This results in increases in credit spreads because it widens the gap between internal and external financing costs.

One important issue that has not yet been addressed in this literature is the possibility of *discontinuous nonlinear asymmetric adjustment process* in the U.S. corporate bond market. The main point of this research is that, if the true adjustment process to the long-run equilibrium path is asymmetric then the conventional linear cointegration analysis suffers from a specification error.[2] The potential damage to financial practitioners will be severe if model builders or traders believe the true data adjustment process is symmetric, and all the trading and hedging strategies are based on the symmetric adjustment assumption.

The current study adopted threshold cointegration methods to study Treasury and corporate bond rates and their corresponding spread behaviors. The first test was to determine whether corporate bond indices and Treasury bonds are cointegrated. If the corporate bond indices are cointegrated with Treasury bonds then the adjustment speeds have pertinent information regarding future economic activities. It was assumed that, if they are cointegrated, the speed of adjustment to the cointegration relation is state dependent. This means that, in a two-regime framework, when the deviation from the long-run equilibrium is positive (negative), the adjustment speed is ρ_1 (ρ_2), where ρ_1 does not need to be equal to ρ_2. The speeds of adjustment were tested to determine whether they are the same if there exists a co-integrating relationship between Treasury bonds and corporate indices. Similarly, in a three-regime environment, it was assumed that Treasury and corporate rates follow a random walk process when they are sufficiently close to the equilibrium level; however, when they are outside the band, they will be mean-reverting to the equilibrium band, possibly with different adjustment speeds. Accordingly, this research documents the threshold

[2] The asymmetry is not the same as the asymmetry discussed by NRM. The focus is more on the asymmetric behaviors of the adjustment speeds to the long-run equilibrium. In NRM, asymmetry comes from different credit spreads before and after three years under a sudden shock to a risk-free rate.

cointegration (and threshold error-correction models) dynamics between Treasury rates and corporate bond rates.

The well-known cost-of-carry model that relates index and index-futures prices was used to illustrate the motivation behind the threshold cointegration approach. In general, futures prices tend to lead index prices. If the futures price is too high relative to the index value, arbitrageurs will buy the stock underlying the index and sell the futures contract. If the futures price is too low, they will do the reverse, i.e., sell the stocks underlying the index and buy the futures contract. Index-futures arbitrageurs only enter into the market if the deviation from the arbitrage relation is sufficiently large to compensate for transaction costs and associated interest rate and dividend risks. In a three-regime threshold cointegration model, the middle regime can be treated as the band around the theoretical futures prices within which arbitrage is not profitable for most arbitrageurs, where both futures and index prices may follow a random walk process without moving closer to each other. Note that, if the band becomes narrower and narrower, a three-regime environment will eventually collapse into a two-regime environment.

The purpose of the current study was also to determine the extent to which threshold cointegration models are able to account for broad time series variations in observed yields of corporate indices for different maturities and different investment grades. By formulating the asymmetric threshold cointegration models advanced by Lo and Zivot (2001), Hansen and Seo (2002), Enders and Siklos (2001), and the traditional symmetric cointegration model pioneered by Engle and Granger (1987), one may be able to identify the most probable adjustment process. Moreover, this study compared the forecast performance of the different models. Out-of-sample forecasts and simulations were be performed and compared for both symmetric and asymmetric models to accomplish the following goals:

- Offer a non-linear, discontinuous, asymmetric alternative to the traditional linear, continuous and symmetric modeling approach.
- Examine credit spread dynamics for different maturities and different investment grades in a bivariate setting of corporate and Treasury rates.
- Determine if corporate and Treasury rates are non-stationary, but cointegrated with nonlinear threshold behavior. If corporate and Treasury rates follow threshold cointegration then the traditional linear cointegration model is misspecified.
- Provide dynamic responses of corporate yields to movements in Treasury rates in a bivariate threshold vector error correction model (TVECM) and a threshold autoregressive (TAR) framework, which will offer more insightful long-run results.
- Apply the same data set as NRM. The current model is very close to NRM, which uses linear cointegration and the impulse response function to shed lights on credit rate dynamics. In this setting, one is able to compare the results for a long-run impact of Treasury shocks on corporate rates, which can have important implications for risk management and bond pricing.

The value of study time series properties of the term structure and corporate yield spreads is rewarding because it can provide better forecasts and understanding of the term structure and corporate yield spreads, which is useful because one can apply them to the following areas of study:

1. Real economic activities which include business cycles theory, stock markets forecasts, inflation forecasts, etc.;

2. Price portfolios comprised of Treasury and corporate bonds;
3. Determine the value of a firm;
4. Price credit derivatives and improving credit management quality;
5. Application to risk management and hedging activities;
6. Price different financial instruments, which include bonds, mortgage backed securities, caps, swaptions, and other financial derivatives;
7. Improve the adequacy of reserves held by banks and insurance companies; and
8. Improve the profitability of trading, the accuracy of current asset pricing, and option pricing models used by securities firms and investment departments.

CHAPTER 2. MOTIVATION AND LITERATURE REVIEW

2.1. Why study the spreads between Treasury and Corporate Bond Indices?

Studying the spread between corporate and Treasury bonds can help one gain a deeper understanding of both the microeconomic level, such as the firm's capital structure, and the macroeconomic level, such as real aggregate economic activities. At the microeconomic firm level, Black and Scholes (1973), and Merton (1974) revealed that, in the study of a firm's value, corporate bonds might be viewed as options on the firm's assets. A great deal of research has focused on the contingent claims (or structural) approach to the pricing of corporate debt (see Duffee (1998), Tauren (1999a) and (1999b)). However, there are two unrealistic assumptions made by previous studies. First, previous models assumed a constant capital structure, which is unrealistic over the life of corporate bonds. Second, previous models predicted that the credit spreads on corporate bonds drift downward over time as the value of the firm's assets drifts upward. However, evidence offered by Duffee (1998) and Tauren (1999b) indicates that credit spreads revert toward long-term averages.

Tauren (1999b) assumed the dynamics of a firm's debt ratio follow a mean-reverting stochastic process. The debt ratio gradually approaches a long-run target ratio. Some predictions from the model are: (a) The model generates credit spreads whose level and dynamics are realistic and comparable to those of actual bonds regardless of investment grades. (b) Credit spreads increase as the firm's target debt ratio increases. The relationship is more important for long-term bonds. The firm's current debt ratio is relatively more important for bonds with short maturities. (c) Credit spreads on corporate bonds decrease at the speed at which the firm adjusts its capital structure toward the long-term target debt ratio.

(d) Credit spreads on long-term corporate bonds exhibit a mean-reversion toward long-term averages determined by the firm's target debt ratio. (e) The mean-reversion in the credit spreads is faster at: high capital structure adjustment speeds, high credit spread levels, and short maturities. (f) Credit spreads are more volatile for short maturities versus long maturities and for high credit spread levels versus low credit spread levels.

Longstaff and Schwartz (1995b), Stevens, Clinebell and Kahl (1998), and Morris, Neal and Rolph (1998) documented that credit spreads calculated from corporate bond yield indices that are mean-reverting. Duffee (1998) and Tauren (1999a) also documented mean-reversion in the credit spreads of individual bonds. The dynamics of the credit spreads on long-term bonds exhibit mean reversion behavior as follows: they drift downward at debt ratios that are above the target debt ratio and upward at debt ratios that are below the target debt ratio. The long-term mean spreads are implied by the target debt ratio. The speed of the mean reversion is positively related to the speed at which the firm adjusts toward the target debt ratio, negatively related to the maturity of the bond, and positively related to the level of the credit spread. The adjustment processes are assumed to be symmetric in both directions and they are conditional on no default occurring.

The firm's debt ratio in Tauren's (1999a) model, reverts toward a long-term target debt ratio. Default is triggered at high values of the debt ratio. The model predicts that the level of the credit spreads of long-term bonds is more sensitive to the firm's target debt ratio than to its current debt ratio. The case is the opposite for bonds with shorter maturities. The credit spreads predicted by the model are also mean-reverting. The model outperforms that of Longstaff and Schwartz (1995a) on bonds from the Boise Cascade Corporation.

At the macroeconomic level, the spread of corporate and Treasury bonds can be related to real economic activities, e.g., business cycles. For example, the current study used corporate bond indices to investigate the dynamic adjustments of corporate bond spreads versus Treasury bonds. As the spread increases, the risk premium increases and the cost of capital increases. As a result, investors may have less confidence in the corporation's earnings, profitability, and future income streams. A resulting direct impact is a hit to the valuation of the corporations, which will eventually reduce the stock prices of the firms and may cause a downgrade of their credit ratings by the rating agencies. One indirect impact will be the wealth effect. Since stock prices have declined, investors perceive they are less affluent and reduce their usual consumption patterns. Eventually, this sequence of activities will lead to a slowdown of the economy.

Another interesting dynamic warranting investigation is the adjustment speeds of the spread to its long-run equilibrium. For example, assume the current spread between a corporate index and Treasury bonds is 4%. Consider two cases: higher spread at 6% versus lower spread at 2%. If an unexpected shock hits the economy and the spread shoots to 6%, investors may lose confidence and their investment will become more risky. Excluding the possibility of any structural changes of the economy, we may compare the situations before and after the shock: if the adjustment process of the spread takes a long time to move back to the "normal" level at 4%, it implies that the economy may sustain a long period of recession. However, if the adjustment is rather quick then the recession is short-lived. Hence, the speed of adjustment plays an important role in the analysis.

On the opposite side, if the shock forces the spread down to 2%, investors have an optimistic outlook for the future economy. It may be one solid economic expansion or it may be one "bubble" expansion (e.g., 1999 Internet new era). For example, if it takes a long time for the spread to adjust back to the "normal" 4% then investors will enjoy a long-term economic expansion. On the other hand, if the adjustment is violently quick then the expansion is a short-life expansion, which implies the bubble bursts rather quickly.

Since the speed of adjustment plays an important role in the reaction to unexpected economic shocks, an immediate testable hypothesis is concerned with whether the speeds of adjustment are the same or not. Casual observations suggest strong co-movements of Treasury bonds rates and corporate indices. Conventional studies usually make the assumption that there exists a linear co-integrating relationship between Treasury bonds and corporate indices, and hence there exists a symmetric adjusting process to the long-run equilibrium. This means that the adjustment speeds are the same either above or below the long-run equilibrium (band). In current literature on credit spreads, there are no serious distinctions to separate the behaviors of adjustment speeds.

2.2. Relationships between Credit Spreads and other Economic and Financial Market Information

In order to estimate and price credit spreads, one needs to understand what kinds of economic forces drive the changes in spreads. In general, spread changes may be related to several risk factors such as interest rates, certain macroeconomic variables, company and industry financial fundamentals, liquidity, time-to-maturity, tax effects, etc. The following sections focus on the relationships of credit spreads to other economic and financial market information.

2.2.1. Credit spreads and treasury rates

Cornell and Green (1991) found that returns on low-grade bonds are much less responsive to changes in Treasury yields than are returns on high-grade bonds. They attributed the result to the shorter duration of low-grade bonds, owing to less restrictive call features and higher coupons. Hence, part of the weak responsiveness of low-grade bond returns to changes in Treasury yields may be driven by a negative correlation between interest rate risk and default risk.

Longstaff and Schwartz (1995a) found a strong negative relationship between Moody's Bond Yield Indices and Treasury bond yields to Treasury yields. A drawback of this result is that Moody's indices are comprised of yields on callable bonds and the strong negative relationship may be driven by variations in the value of the call options.

Iwanowski and Chandra (1995) examined the relationship between Treasury yields and yield spreads of non-callable bonds during 1980s and 1990s. They found a small negative relationship between the levels of the Treasury yield and yield spreads, and no significant relationship between the Treasury term structure slope and spreads. Duffee (1996) argued that, because Iwanowski and Chandra (1995) used refreshed yield indices,[3] the result may underestimate the responsiveness of yield spreads to Treasury yields, because it cannot capture changes in yields on bonds that were upgraded or downgraded during that period.

[3] A bond index will change its rating due to the bonds it holds, either downgraded between periods t and t + 1 or that fallout of the maturity range between t and t + 1. A "refreshed" yield index is an index that holds credit ratings fixed over time by changing its portfolio of bonds.

2.2.2. Credit spreads and the business cycle

Research linking the shape of the Treasury term structure to future variations in the business cycle, measured by NBER dating or the growth rate of Gross Domestic Product (GDP), includes Stock and Watson (1989), Chen (1991), Estrella and Hardouvelis (1991), and Estrella and Mishkin (1996). One well-known stylized fact is that low yields and a steeply sloped term structure both correspond to future economic expansion. Duffee (1996) conjectured that: "*low yields and a steep slope forecast future high economic growth. When the growth arrives, yields rise and the slope falls. Low yields (or a steep slope) forecast both future growth and future increases in yields (or future decreases in the slope)*". Duffee offered the following interpretation: "*…when Treasury yields rise, the economy is expanding, firms are better off, and default probabilities fall. But when the slope of the yield curve steepens, the economy is contracting, so default probabilities should be rising, which should widen the yield spreads*". However, this argument does not earn support from the empirical study regarding the slope coefficients which are inconsistent with such a standard business cycle scenario (see Duffee, 1996, Table 2).

2.2.3. Credit spreads and the pricing of corporate bonds and value of the firm

Black and Scholes (1973), and Merton (1974) revealed that corporate bonds might be viewed as options on the firm's assets in the study of the firm's value. This contingent claims (structural) approach uses company-specific information and treats debt as a contingent claim (option) on the firm's value. However, there are two unrealistic assumptions made by previous studies. First, previous models assume a constant capital structure, which is unrealistic over the life of corporate bonds. Second, previous models predict the credit spreads on corporate bonds drift downward over time as the value of the firm's assets drifts upward. Empirical evidence offered by Duffee (1998) and Tauren (1999b) indicate that credit spreads revert toward long-term averages.

Credit spreads are more volatile at high levels than at low levels of the credit spread. Volatilities are positively related to term-to-maturity at short maturities, but are a decreasing function of term-to-maturity at more typical maturities. In Tauren's (1999a, b) model, the correlation between the changes in the credit spreads and the default-free interest rate is positive, because interest rates tend to be negatively correlated with asset values. However, Kwan (1996), Longstaff and Schwartz (1995a), and Duffee (1996) documented that changes in credit spreads and default-free interest rates are negatively correlated. This correlation is negatively related to the credit quality of the bond. Tauren (1999a) concluded that there exists an empirical relationship between the default risk in corporate bonds and interest rates that are not taken into account by the model.

2.2.4. Credit spreads and default rates, credit derivatives, and risk management

Fons, Carty, and Kaufman (1994) found that future default rates on bonds rated by Moody's are positively correlated with forecasts of future GDP growth. Longstaff and Schwartz (1995a) showed by using a non-callable zero-coupon corporate bond model, that variations over time in default risk are accompanied by variations in yield spreads. Hence, patterns in the behavior of spreads can be used to make inferences about the relationship between default risk and interest rates.

Jarrow and Turnbull (1995a, b) developed a reduced-form model to study the firm's value, in contrast to the contingent claims approach pioneered by Black and Scholes (1973), and Merton (1974). The reduced-form approach bypasses issues related to firm valuation and works directly with market information. Their model can infer conditional Martingale probabilities of default from the term structure of credit spreads. Economic theory states that market and credit risk are inherently interrelated. This interdependence property between market and credit risk can affect the asset allocation decisions of economic capital. It can also affect the performance evaluation of determining risk-adjusted return on capital (RAROC) among different investment avenues.

The dynamics between market and credit risk are straightforward: if the market value of the firm's assets changes drastically—this generates market risk—which will affect the probability of default rate—which will generate credit risk. Conversely, if the probability of default rate unexpectedly changes—this generates credit risk—which will affect the market value of the firm—which will generate market risk.

Jarrow and Turnbull (1995a, b) priced credit derivatives off the observable term structures of interest rates for different credit classes. They used observable market data—credit spreads—to infer the market's assessment of the bankruptcy process and then price credit risk derivatives. Jarrow and Turnbull (2000) modeled the default process as a multi-factor Cox process; the intensity function is assumed to depend upon different state variables. Modeling the intensity function as a Cox process enabled Jarrow and Turnbull to model the empirical observation that the credit spread depends on both the default-free term structure and an equity index. Referring to empirical evidence in this area, Shane (1994) stated that: *"returns on high yield bonds have a higher correlation with the return on an equity index than low yield bonds and a lower correlation with the return on a Treasury bond index than lower yield bonds"*. As in Longstaff and Schwartz (1995a, b), and Das and Tufano (1996), Duffee (1998), Jarrow and Turnbull (2000) stated: *"the changes in credit spreads and changes in default free interest rates being negatively correlated, and the estimated coefficients increase in absolute magnitude as the credit quality decreases irrespective of maturity"*. The model developed by Jarrow and Turnbull (2000) can be used for risk management and hence it is possible to price portfolios of corporate bonds and credit derivatives in a more consistent way.

2.2.5. Credit spreads and the equity market

Chen (1991) found that aggregate corporate yield spreads are linked to stock returns, which, in turn, are linked to future GDP growth. Duffee (1996) noted that stock returns move inversely with corporate bond yield spreads. The relationship is stronger for lower-rated bonds. A 10% increase in the S&P 500 corresponds to a 20 basis point decrease in Baa-rated yield spreads (regardless of maturity). The same increase in stock prices corresponds to a 10 basis point decrease in A-rated yield spreads and small decreases in higher-rated yield spreads. Duffee (1996) also showed that an increase in yield spreads of 100 basis points correspond to a 2.13% decline in the growth rate of the GDP. Duffee's evidence provided no apparent pattern across maturities or credit ratings.

However, there is a troubling aspect of Duffee's equation (12). The inclusion of stock returns had no important explanatory power on the level of the term structure. As Duffee (1996) noted: "*If these variations in yield spreads are not driven by variations in default risk, then one cannot use the behavior of corporate bond yields to parameterize models of credit risk for the purposes of pricing other types of default-risky instruments*". Duffee pointed out that equation (12) is almost misspecified because it assumes a simple linear relationship between stock return and yield spreads. Duffee (1996) suggested one should at least consider the possibility that the variation between yield spreads and the Treasury yields is unrelated to default risk.

2.2.6. Credit spreads and liquidity premium

Grinblatt (1995) argued that yield spreads on short-term corporate instruments are more likely driven by the liquidity of Treasury instruments than the risk of default. It is plausible to believe that the value of liquidity varies with the Treasury term structure. This hypothesis can be tested with spreads on default-free instruments such as government agency bonds (Ginny Mae and Fannie Mae). This is one possible future research topic suggested by Duffee (1996).

2.2.7. Credit spreads and a trader's story of supply and demand

Traders postulate that when bond yields (both Treasury and corporate) fall, firms respond by issuing more bonds, but the Treasury does not do likewise. The relative increase in the supply of corporate bonds lowers the price of corporate debt relative to Treasury debt, and hence widens the yield spread. This is one more future research topic suggested by Duffee (1996).

2.3. Literature review on the empirical studies using Threshold Cointegration

Ghosh (1993) and Brenner and Kroner (1995) have shown that, under certain conditions, futures and spot prices are cointegrated. This implies an error-correction model for the returns in which the futures and index returns are explained by past futures and index returns, and the deviations (i.e., error-correction term or mispricing error) from the no-arbitrage relationship in the previous period. It has been documented that futures prices tend to lead index prices (e.g., Stoll and Whaley (1990), and Chan (1992)). Hence, deviations from the no-arbitrage relationship will occur when the futures react to news first, followed by the index. Such a pattern is reflected by the significant impact of past futures returns on the current index return and by a significant error-correction term. Martens, Kofman and Vorst (1998) investigated the index-futures arbitrageurs' trading strategy and concluded that arbitrageurs only enter into the market if the deviation from the arbitrage relation is large enough to offset the transaction costs and the associated interest rate and dividend risks.

Based on the Balke and Fomby (1997) framework, Martens, Kofman and Vorst (1998) used a threshold autoregression model to estimate the band around the theoretical futures price within which arbitrage is not profitable or at least not for a large group of arbitrageurs. The data set used in their study included the S&P 500 index and the matching index-futures contract maturing in June and December 1993. The S&P 500 index was calculated every 15 seconds during the opening hours of the New York Stock Exchange (NYSE); for the index-futures, traded at the Chicago Mercantile Exchange (CME), and transaction prices were available with a time stamp to the nearest second. Martens et al. (1998) showed that by combining a threshold effect, with an error-correction model, the impact of the cointegrating residual (i.e., mispricing error) is increasing with the magnitude of that error and the information effect of lagged futures returns on index returns is significantly larger when the mispricing error is negative. They also showed that the U.S. markets respond to arbitrage opportunities in just a few minutes. They also provided an estimate of the error-correction model in each regime.

Balke and Wohar (1998) examined the dynamics of deviations from covered interest parity using daily data on the UK/US spot and forward exchange rates and their interest rates over the period January 1974 to September 1993. They found a substantial number of instances during the sample period in which deviations from the covered interest parity condition exceeded the transaction costs, implying possible arbitrage profit opportunities. In their analysis, although most of the implied profit opportunities are relatively small, there is evidence of some large deviations from covered interest parity. Balke and Wohar (1998) estimated a threshold autoregression model such that the dynamic behavior of deviations from covered interest parity is different outside the transaction costs band than inside the band. Their results indicated that inside the transaction band the impulse response functions are nearly symmetric; but impulse response functions are asymmetric outside the bands. Their findings also suggested that deviations from covered interest parity that are outside the transaction costs band show less persistence than those that lie inside the band.

There are also applications of the threshold cointegration model to the area of agricultural economics. For example, Goodwin and Holt (1999) utilized the Balke and Fomby (1997) threshold cointegration method to investigate price linkages among producers, wholesale, and retail marketing channels in the U.S. beef market. They also evaluated the dynamics of the time paths of price adjustments to shocks at each level in the U.S. beef marketing channels. The data set included three series of weekly beef prices observed from January 1981 through the first week of March 1998, giving a total of 897 observations.

Their results confirmed previous findings. In particular, the transmission of shocks is unidirectional with information flowing up the marketing channel from farm to wholesale to retail markets, but not in the opposite direction. Results show that farm markets do adjust to wholesale market shocks. The effects of retail market shocks are confined to retail markets. Their evaluation of nonlinear impulse response functions suggests there may be asymmetric adjustments in response to new information. However, these differences are modest and may not be economically significant. Their results also suggest that the responsiveness to price shocks has increased in recent years. This finding might suggest markets have become more efficient in transmitting information through vertical marketing channels. Goodwin and Harper (2000) conducted a parallel analysis to Goodwin and Holt (1999) in a study of price dynamics and transmission of shocks through marketing channels in the U.S. pork sector. In a similar setting of a threshold cointegration model, they also confirmed that price adjustment patterns are unidirectional and information tends to flow from farm, to wholesale, to retail markets.

The next chapter introduces asymmetric threshold cointegration models advanced by Lo and Zivot (2001), Hansen and Seo (2002), and Enders and Siklos (2001) to capture more insights of credit spread behaviors. Lo and Zivot (2001) provided a multivariate procedure for modeling threshold cointegration relationships. Based on Granger's Representation Theorem, the setting is a threshold vector error-correction model (TVECM) instead of a single-equation threshold cointegration model such as Balke and Fomby (1997), and Enders and Siklos (2001). Their model may capture both the long-term equilibrium relationship and the short-term disequilibrium adjustment process towards the long-term equilibrium. Hansen and Seo (2002) proposed a formal test procedure for threshold cointegration. They offered an algorithm to estimate model parameters. The Hansen-Seo approach is a vector error correction model with only one cointegrating vector and with one build-in threshold effect in the error-correction term in a two-regime environment. Enders and Siklos (2001) pointed out that, if the adjustment process is indeed asymmetric, then the Engle-Granger cointegration test is misspecified and the error-correction mechanism is unable to capture the actual adjustment process. They suggested an alternative specification of the error-correction model in the form of threshold autoregressive framework. They developed asymmetric cointegration tests by incorporating threshold autoregressive (TAR) and momentum threshold autoregressive (M-TAR) adjustments into the unit-root tests of the residuals of the cointegration regression.

CHAPTER 3. METHODOLOGIES

3.1. Introduction

Threshold effects occur when larger shocks bring about a different response than do smaller shocks. The resulting dynamic responses will have a nonlinear nature, because they involve various combinations of adjustments from different regimes defined by different thresholds. These aspects are also recognized as significant characteristics of regimes switching models.

Balke and Fomby (1997) focused on the fact that equilibrium errors can have threshold behavior. In their model, the deviation from long-run equilibrium follows discontinuous adjustment to a long-run equilibrium band, or put it in another way, the equilibrium error follows a threshold autoregressive process (TAR) that is mean-reverting outside a band but is a unit root process inside the band. Three types of threshold autoregressive (TAR) models are considered for the equilibrium error process z_t; where z_t is obtained from the standard single equation cointegration regression: $y_t + \alpha x_t = z_t$. Balke and Fomby (1997) formulated: (a) the equilibrium threshold (EQ-TAR) model, (b) the band threshold (Band-TAR) model, and (c) the return-drift threshold (RD-TAR) model. The EQ-TAR model has the characteristic that the process tends towards a long-term equilibrium point (for example, zero) when equilibrium error is outside the region $[-\theta, \theta]$. The Band-TAR model has the property that the process returns to an equilibrium band $[-\theta, \theta]$ rather than to an equilibrium point as defined in EQ-TAR. The RD-TAR model is where unit root process is present in all three regimes, but the drift parameters tend to move the process back to the band when the equilibrium error is outside the band. A two-step approach is offered to examine this threshold cointegration behavior. The first step is to test the global cointegration behavior of the variable of interest. Balke and Fomby (1997) assert that most standard tests for cointegration, even if the errors follow a TAR process, e.g., Engle-Granger's (1987) single equation approach and Johansen's (1988, 1991) full information maximum likelihood approach, are able to detect the stationarity of the equilibrium error. The second step is to test the local behavior, i.e., to test for nonlinearity, and, especially, for threshold nonlinearity in the equilibrium error process. Using an arranged autoregression for a given potential threshold values, the threshold autoregression is estimated by ordinary least squares and the sum of squared errors are calculated. The estimated thresholds are the ones that minimize the sum of squared errors. Balke and Fomby formed their hypothesis of no structural change versus the alternative of two structural changes. The test statistic is a sup-Wald (maximum Wald) statistic over all possible threshold values.

3.2. Model 1: Lo-Zivot Threshold Cointegration Model

Although Balke and Fomby (1997) provide an excellent way to model threshold behavior, their approach suffers from at least two drawbacks. First, Balke and Fomby (1997) focused on the univariate cointegrating residual behavior, so their method is unable to investigate the threshold behavior in a multivariate setting. In general, the benefit of multivariate modeling is that it enables one to investigate the dynamic adjustment of individual series more efficiently and it is also easier to uncover the overall dynamics of the whole multivariate system. Lo and Zivot (2001) provided a procedure for modeling multivariate threshold cointegration relationships. With the introduction of a threshold vector error correction model (TVECM), Lo and Zivot (2001) asserted that the threshold error correction model could capture the long-run equilibrium relationship as well as the short-term disequilibrium adjustment process to the long-run equilibrium.

Second, Balke and Fomby (1997) did not provide a specification test for their imposing TAR models, i.e., the model selection process in Balke and Fomby (1997) is ad hoc. The current research applied a specification test offered by Hansen (1997, 1999) and advocated by Lo and Zivot (2001) to determine which TAR model is appropriate to capture threshold cointegration relationships for Treasury and corporate bond rates. In particular, the current study focused on investigating yield spread behavior between different investment grade and different maturity indices in U.S. Treasury and corporate bonds. In summary, a multivariate threshold vector error correction model allowing discontinuous adjustment relative to the thresholds, nonlinear adjustments to the long-run equilibrium, and asymmetric adjusting speeds to the long-run equilibrium was estimated and used to evaluate the dynamic time paths of yield spread adjustments to U.S. Treasury and corporate bond indices.

3.2.1. A bivariate vector error correction model (VECM)

Consider a bivariate vector autoregressive (VAR) model, where X_t is a 2×1 vector with $X_t' = (x_{1t}, x_{2t})$, for example, x_{1t} is the corporate bond rate, and x_{2t} is the Treasury rate. Then

$$X_t = A_0 + \sum_{i=1}^{k} A_i X_{t-i} + \varepsilon_t, \tag{3.1}$$

where ε_t is a 2×1 white noise process, k is the order of autoregressive terms, A_0 is a 2×1 parameter vector, and A_i's are 2×2 parameter matrices. This equation can be rewritten as:

$$\Delta X_t = A_0 + \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t, \tag{3.2}$$

where $\Pi = -\left(\sum_{i=1}^{k} A_i - I_2\right)$, and $\Gamma_i = -\sum_{\ell=i+1}^{k} A_\ell$, for i = 1, 2, …, k-1.

If elements of X_t are I(1) and are cointegrated with a normalized cointegrating vector $\beta' = (1, -\beta_2)$ then equation (3.2) can be expressed as a vector error-correction model (VECM) as follows:

$$\Delta X_t = A_0 + \gamma\beta' X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \varepsilon_t, \tag{3.3}$$

where

$$\Pi = \gamma\beta' = \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} (1, -\beta_2) = \begin{pmatrix} \gamma_1 & -\beta_2\gamma_1 \\ \gamma_2 & -\beta_2\gamma_2 \end{pmatrix}. \tag{3.4}$$

The γ represents the speeds of adjustment while $\beta' X_{t-1}$ denotes the error-correction terms, or the cointegrating residuals.

3.2.2. The Band-TVECM (Band-Threshold Vector Error Correction Model)

Although conventional VAR and VECM have modeling power regarding numerous economic and financial phenomena, they can only model linear relationships. In recent time series econometrics developments, both the threshold autoregression (TAR) and threshold vector error correction model (TVECM) have overcome this drawback. The TAR and TVECM have the strength of modeling nonlinear and discontinuous phenomenon. Consider a simple three-regime bivariate TVECM for the threshold cointegrating relationship of the Treasury and corporate bond rates. The bivariate threshold vector autoregressive (TVAR) model for X_t is:

$$X_t = \left[A_0^{(1)} + \sum_{i=1}^{k} A_i^{(1)} X_{t-i} + \varepsilon_t^{(1)} \right] I_{1t}(z_{t-d} \le c^{(1)})$$
$$+ \left[A_0^{(2)} + \sum_{i=1}^{k} A_i^{(2)} X_{t-i} + \varepsilon_t^{(2)} \right] I_{2t}(c^{(1)} < z_{t-d} \le c^{(2)}) \tag{3.5}$$
$$+ \left[A_0^{(3)} + \sum_{i=1}^{k} A_i^{(3)} X_{t-i} + \varepsilon_t^{(3)} \right] I_{3t}(z_{t-d} > c^{(2)}),$$

where $\varepsilon_t^{(j)}$'s are bivariate vector white noise processes, k is the autoregressive order, $A_0^{(j)}$'s are 2 × 1 parameter vectors, and $A_i^{(j)}$'s are 2 × 2 parameter matrices for regime j = 1, 2, 3, and for lag i = 1, 2, …, k; z_{t-d} is called the threshold variable; d is called the delay parameter, which is positive and usually less than or equal to the lag length k. In general, $-\infty = c^{(0)} < c^{(1)} < c^{(2)} < c^{(3)} = \infty$ and

$$I_{jt}(c^{(j-1)} < z_{t-d} \le c^{(j)}) = \begin{cases} 1, \text{ if } c^{(j-1)} < z_{t-d} \le c^{(j)}, j = 1,2,3, \\ 0, \text{ otherwise.} \end{cases} \tag{3.6}$$

If elements of X_t are I(1) and they are cointegrated then equation (3.5) can be expressed as a TVECM as follows:

$$\Delta X_t = \left[A_0^{(1)} + \Pi^{(1)} X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i^{(1)} \Delta X_{t-i} + \varepsilon_t^{(1)} \right] I_{1t}(z_{t-d} \le c^{(1)})$$
$$+ \left[A_0^{(2)} + \Pi^{(2)} X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i^{(2)} \Delta X_{t-i} + \varepsilon_t^{(2)} \right] I_{2t}(c^{(1)} < z_{t-d} \le c^{(2)}) \tag{3.7}$$
$$+ \left[A_0^{(3)} + \Pi^{(3)} X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i^{(3)} \Delta X_{t-i} + \varepsilon_t^{(3)} \right] I_{3t}(z_{t-d} > c^{(2)}),$$

where $\Pi^{(j)} = -\left(\sum_{i=1}^{k} A_i^{(j)} - I_2 \right)$, and $\Gamma_i^{(j)} = -\sum_{\ell=i+1}^{k} A_\ell^{(j)}$ for regime j = 1, 2, 3, and i = 1, 2, …, k-1.

Furthermore, if elements of X_t are cointegrated with a common (across regime) normalized cointegrating vector $\beta' = (1, -\beta_2)$ and if the error terms $\varepsilon_t^{(j)}$ share the same variance-covariance structure then the TVECM may be written as:

$$\begin{aligned}\Delta X_t = &\left[A_0^{(1)} + \gamma^{(1)}\beta' X_{t-1} + \sum_{i=1}^{k-1}\Gamma_i^{(1)}\Delta X_{t-i}\right] I_{1t}(z_{t-d} \le c^{(1)}) \\ &+ \left[A_0^{(2)} + \gamma^{(2)}\beta' X_{t-1} + \sum_{i=1}^{k-1}\Gamma_i^{(2)}\Delta X_{t-i}\right] I_{2t}(c^{(1)} < z_{t-d} \le c^{(2)}) \\ &+ \left[A_0^{(3)} + \gamma^{(3)}\beta' X_{t-1} + \sum_{i=1}^{k-1}\Gamma_i^{(3)}\Delta X_{t-i}\right] I_{3t}(z_{t-d} > c^{(2)}) + \varepsilon_t,\end{aligned} \tag{3.8}$$

where

$$\gamma^{(j)}\beta' = \Pi^{(j)} = \begin{pmatrix}\gamma_1^{(j)} \\ \gamma_2^{(j)}\end{pmatrix}(1, -\beta_2) = \begin{pmatrix}\gamma_1^{(j)} & -\beta_2\gamma_1^{(j)} \\ \gamma_2^{(j)} & -\beta_2\gamma_2^{(j)}\end{pmatrix}, \tag{3.9}$$

and j = 1, 2, 3.

Note that although the three regimes share a common cointegrating vector $\beta' = (1, -\beta_2)$, the speeds of adjustment $\gamma^{(j)\prime} = (\gamma_1^{(j)}, \gamma_2^{(j)})$ are regime specific. For example, we may observe that $\gamma_1^{(1)} \ne \gamma_1^{(3)}$ or $\gamma_2^{(2)} \ne \gamma_2^{(3)}$.

The simplest form for the TVECM occurs when k = 1 in equation (3.8), so that all lag difference terms drop out of the equation. In this situation, the cointegrating residual $\beta' X_t$ follows a regime specific AR(1) process or threshold autoregressive (TAR) process:

$$\beta' X_t = \delta^{(j)} + \rho^{(j)}\beta' X_{t-1} + \eta_t^{(j)},$$

with

$$\rho^{(j)} = 1 + \beta'\gamma^{(j)} = 1 + \gamma_1^{(j)} - \beta_2\gamma_2^{(j)},$$

where $\delta^{(j)} = \beta' A_0^{(j)}$ and $\eta_t^{(j)} = \beta' \varepsilon_t^{(j)}$.[4] Furthermore, $\beta' X_t$ is stable within each regime, if the stability condition $|\rho^{(j)}| = |1 + \gamma_1^{(j)} - \beta_2 \gamma_2^{(j)}| < 1$ holds for each regime. Again, in equation (3.8), with k = 1, the equations will be:

$$\begin{aligned} \Delta X_t = & \left[A_0^{(1)} + \gamma^{(1)} \beta' X_{t-1}\right] I_{1t}(z_{t-d} \le c^{(1)}) + \left[A_0^{(2)} + \gamma^{(2)} \beta' X_{t-1}\right] I_{2t}(c^{(1)} < z_{t-d} \le c^{(2)}) \\ & + \left[A_0^{(3)} + \gamma^{(3)} \beta' X_{t-1}\right] I_{3t}(z_{t-d} > c^{(2)}) + \varepsilon_t . \end{aligned} \tag{3.10}$$

It is easier to capture the long-run equilibrium relationship if equation (3.10) is rewritten in the following form:

$$\begin{aligned} \Delta X_t = & \gamma^{(1)} \left[\beta' X_{t-1} - \mu^{(1)}\right] I_{1t}(z_{t-d} \le c^{(1)}) + \gamma^{(2)} \left[\beta' X_{t-1} - \mu^{(2)}\right] I_{2t}(c^{(1)} < z_{t-d} \le c^{(2)}) \\ & + \gamma^{(3)} \left[\beta' X_{t-1} - \mu^{(3)}\right] I_{3t}(z_{t-d} > c^{(2)}) + \varepsilon_t . \end{aligned} \tag{3.11}$$

More specifically, one has:

$$\Delta x_{1t} = \begin{cases} \gamma_1^{(1)} \left[x_{1t-1} - \beta_2 x_{2t-1} - \mu^{(1)}\right] + \varepsilon_{1t}^{(1)}, & \text{if } z_{t-d} \le c^{(1)}, \\ \gamma_1^{(2)} \left[x_{1t-1} - \beta_2 x_{2t-1} - \mu^{(2)}\right] + \varepsilon_{1t}^{(2)}, & \text{if } c^{(1)} < z_{t-d} \le c^{(2)}, \\ \gamma_1^{(3)} \left[x_{1t-1} - \beta_2 x_{2t-1} - \mu^{(3)}\right] + \varepsilon_{1t}^{(3)}, & \text{if } z_{t-d} > c^{(2)}, \end{cases}$$

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[4] Set k = 1 in equation (3.8) to obtain:

$$\begin{aligned} \Delta X_t = X_t - X_{t-1} = & \left[A_0^{(1)} + \gamma^{(1)} \beta' X_{t-1}\right] I_{1t}(z_{t-d} \le c^{(1)}) + \left[A_0^{(2)} + \gamma^{(2)} \beta' X_{t-1}\right] I_{2t}(c^{(1)} < z_{t-d} \le c^{(2)}) \\ & + \left[A_0^{(3)} + \gamma^{(3)} \beta' X_{t-1}\right] I_{3t}(z_{t-d} > c^{(2)}) + \varepsilon_t . \end{aligned}$$

Multiply both sides by β', then move $\beta' X_{t-1}$ to the right-hand side, will obtain:

$$\begin{aligned} \beta' X_t = \beta' X_{t-1} + & \left[\beta' A_0^{(1)} + \beta' \gamma^{(1)} \beta' X_{t-1}\right] I_{1t}(z_{t-d} \le c^{(1)}) + \left[\beta' A_0^{(2)} + \beta' \gamma^{(2)} \beta' X_{t-1}\right] I_{2t}(c^{(1)} < z_{t-d} \le c^{(2)}) \\ & + \left[\beta' A_0^{(3)} + \beta' \gamma^{(3)} \beta' X_{t-1}\right] I_{3t}(z_{t-d} > c^{(2)}) + \beta' \varepsilon_t . \end{aligned}$$

Split $\beta' X_{t-1}$ and ε_t to each regime to then obtain:

$$\begin{aligned} \beta' X_t = & \left[\beta' A_0^{(1)} + \beta' X_{t-1} + \beta' \gamma^{(1)} \beta' X_{t-1} + \beta' \varepsilon_t^{(1)}\right] I_{1t}(z_{t-d} \le c^{(1)}) \\ & + \left[\beta' A_0^{(2)} + \beta' X_{t-1} + \beta' \gamma^{(2)} \beta' X_{t-1} + \beta' \varepsilon_t^{(2)}\right] I_{2t}(c^{(1)} < z_{t-d} \le c^{(2)}) \\ & + \left[\beta' A_0^{(3)} + \beta' X_{t-1} + \beta' \gamma^{(3)} \beta' X_{t-1} + \beta' \varepsilon_t^{(3)}\right] I_{3t}(z_{t-d} > c^{(2)}). \end{aligned}$$

Collect terms to get:

$$\begin{aligned} \beta' X_t = & \left[\beta' A_0^{(1)} + (1 + \beta' \gamma^{(1)}) \beta' X_{t-1} + \beta' \varepsilon_t^{(1)}\right] I_{1t}(z_{t-d} \le c^{(1)}) \\ & + \left[\beta' A_0^{(2)} + (1 + \beta' \gamma^{(2)}) \beta' X_{t-1} + \beta' \varepsilon_t^{(2)}\right] I_{2t}(c^{(1)} < z_{t-d} \le c^{(2)}) \\ & + \left[\beta' A_0^{(3)} + (1 + \beta' \gamma^{(3)}) \beta' X_{t-1} + \beta' \varepsilon_t^{(3)}\right] I_{3t}(z_{t-d} > c^{(2)}) \\ = & \ \delta^{(j)} + \rho^{(j)} \beta' X_{t-1} + \eta_t^{(j)} . \end{aligned}$$

and

$$\Delta x_{2t} = \begin{cases} \gamma_2^{(1)}\left[x_{1t-1} - \beta_2 x_{2t-1} - \mu^{(1)}\right] + \varepsilon_{2t}^{(1)}, & if\ z_{t-d} \le c^{(1)}, \\ \gamma_2^{(2)}\left[x_{1t-1} - \beta_2 x_{2t-1} - \mu^{(2)}\right] + \varepsilon_{2t}^{(2)}, & if\ c^{(1)} < z_{t-d} \le c^{(2)}, \\ \gamma_2^{(3)}\left[x_{1t-1} - \beta_2 x_{2t-1} - \mu^{(3)}\right] + \varepsilon_{2t}^{(3)}, & if\ z_{t-d} > c^{(2)}. \end{cases}$$

The magnitudes and signs of the γ's will provide fruitful information regarding the equilibrium relationships. Equation (3.11) offers the regime-specific means $\mu^{(j)}$, which is calculated as:

$$\mu^{(j)} = -\frac{\beta' A_0^{(j)}}{\beta' \gamma^{(j)}} = -\frac{A_{0,1}^{(j)} - \beta_2 A_{0,2}^{(j)}}{\gamma_1^{(j)} - \beta_2 \gamma_2^{(j)}} = \frac{\delta^{(j)}}{1 - \rho^{(j)}}, \tag{3.12}$$

where $A_0^{(j)}{}' = (A_{0,1}^{(j)}, A_{0,2}^{(j)})$, and $\beta' = (1, -\beta_2)$. It is also possible to eliminate the regime specific drift in X_t through the restriction:

$$A_0^{(j)} = -\gamma^{(j)} \mu^{(j)} \tag{3.13}$$

where $\mu^{(j)}$ is calculated by (3.12). Note, we may rewrite equation (3.11) as follows with $z_{t-1} = \beta' X_{t-1} = x_{1t-1} - \beta_2 x_{2t-1}$:

$$\Delta X_t = \begin{cases} \gamma^{(1)}\left[z_{t-1} - \mu^{(1)}\right] + \varepsilon_t, & if\ z_{t-d} \le c^{(1)}, \\ \gamma^{(2)}\left[z_{t-1} - \mu^{(2)}\right] + \varepsilon_t, & if\ c^{(1)} < z_{t-d} \le c^{(2)}, \\ \gamma^{(3)}\left[z_{t-1} - \mu^{(3)}\right] + \varepsilon_t, & if\ z_{t-d} > c^{(2)}. \end{cases} \tag{3.14}$$

Consider the case of d = 1, $\gamma^{(2)} = 0$ and $A_0^{(2)} = 0$ in equation (3.14). This is the Band-TVECM structure, which is the most popular form in threshold cointegrating applications:

$$\Delta X_t = \begin{cases} \gamma^{(1)}\left[z_{t-1} - \mu^{(1)}\right] + \varepsilon_t, & if\ z_{t-1} \le c^{(1)}, \\ \varepsilon_t, & if\ c^{(1)} < z_{t-1} \le c^{(2)}, \\ \gamma^{(3)}\left[z_{t-1} - \mu^{(3)}\right] + \varepsilon_t, & if\ z_{t-1} > c^{(2)}. \end{cases} \tag{3.15}$$

Again, the stability conditions must hold for the outer regimes, i.e., $|\rho^{(j)}| = |1+\gamma_1^{(j)} - \beta_2\gamma_2^{(j)}| < 1$, for j = 1 and 3. The interpretation of previous model is: if the cointegrating residual (the error-correction term) $z_{t-1} = \beta' X_{t-1}$ lies within the inner band $[c^{(1)}, c^{(2)}]$ then X_t behaves like a random walk process without the drift, i.e., ΔX_t has no tendency reverting to any long-term equilibrium; if z_{t-1} is less than $c^{(1)}$ then z_t reverts to the regime specific mean $\mu^{(1)}$ with adjustment coefficient $\rho^{(1)}$ while ΔX_t adjusts with speed of adjustment vector $\gamma^{(1)}$; if z_{t-1} is greater than $c^{(2)}$ then z_t reverts to the regime specific mean $\mu^{(3)}$ with adjustment coefficient $\rho^{(3)}$ and ΔX_t adjusts with speed of adjustment vector $\gamma^{(3)}$. One may expect $\gamma_i^{(3)} \leq 0$, $\gamma_i^{(1)} > 0$, for i = 1, 2, because of the force of the error correcting toward the long-term equilibrium.

If the regime specific means of the cointegrating residual z_t are equal to the nearby threshold values: $\mu^{(1)} = c^{(1)}, \mu^{(3)} = c^{(2)}$ then (3.15) may be written as:

$$\Delta X_t = \begin{cases} \gamma^{(1)}\left[z_{t-1} - c^{(1)}\right] + \varepsilon_t, & \text{if } z_{t-1} \leq c^{(1)}, \\ \varepsilon_t, & \text{if } c^{(1)} < z_{t-1} \leq c^{(2)}, \\ \gamma^{(3)}\left[z_{t-1} - c^{(2)}\right] + \varepsilon_t, & \text{if } z_{t-1} > c^{(2)}. \end{cases} \tag{3.16}$$

It is called the "continuous" model. Furthermore, the "symmetric" threshold model arises when the threshold values are symmetric against the origin, i.e., $c^{(2)} = -c^{(1)} = c$, then one has:

$$\Delta X_t = \begin{cases} \gamma^{(1)}\left[z_{t-1} + c\right] + \varepsilon_t, & \text{if } z_{t-1} \leq -c, \\ \varepsilon_t, & \text{if } -c < z_{t-1} \leq c, \\ \gamma^{(3)}\left[z_{t-1} - c\right] + \varepsilon_t, & \text{if } z_{t-1} > c. \end{cases} \tag{3.17}$$

If $\mu^{(1)} = \mu^{(3)} = 0$, then one has the EQ-TVECM:

$$\Delta X_t = \begin{cases} \gamma^{(1)} z_{t-1} + \varepsilon_t, & \text{if } z_{t-1} \le c^{(1)}, \\ \varepsilon_t, & \text{if } c^{(1)} < z_{t-1} \le c^{(2)}, \\ \gamma^{(3)} z_{t-1} + \varepsilon_t, & \text{if } z_{t-1} > c^{(2)}. \end{cases} \tag{3.18}$$

3.2.3. Hansen's procedures for testing linearity

Once it has been determined that X_t is cointegrated with known cointegrating vector β, the next step is to determine if the dynamics in the cointegrating relationship are linear or exhibits threshold nonlinearity. Hansen (1997,1999) developed a method for testing the null hypothesis of linearity versus the alternative of a TAR(*m*) model, where *m* denotes the number of regimes, based on nested hypothesis tests. Consider the TAR(m) model for $z_{t-1} = \beta' X_{t-1}$:

$$z_t = \delta^{(j)} + \rho^{(j)} z_{t-1} + \eta_t^{(j)},\ j=1, 2, \dots, m. \tag{3.19}$$

A linear autoregressive model (i.e., TAR(l)) results under the restrictions that $\delta^{(j)} = \delta$ and $\rho^{(j)} = \rho$, $\forall j$. Hansen's linearity test is a test of the null hypothesis of TAR(l) against the alternative of TAR(*m*) for some $m > 1$ using a sup-*F* (or sup-Wald) test constructed from the supremum over possible threshold values of the F-statistic:

$$F_{1,m} = T\left(\frac{S_1 - S_m}{S_m}\right), \tag{3.20}$$

where S_l and S_m denote the sum of squared residuals from the estimation of a TAR(1) model and a TAR(*m*) model, respectively. Hansen provided a simple bootstrap procedure to compute *p*-values for this test.

Hansen's method for testing linearity in univariate TAR models based on nested hypothesis tests can be easily extended to test linearity in multivariate TVECMs. To test the null hypothesis of a linear VECM against the alternative of a TVECM(m) for some $m > 1$, the test statistic is the sup-LR statistic (which is asymptotically equivalent to the sup-Wald) constructed from:

$$LR_{1,m} = T(\ln(|\hat{\Sigma}|) - \ln(|\hat{\Sigma}_m(\hat{c},\hat{d})|)) \tag{3.21}$$

where $\hat{\Sigma}$ and $\hat{\Sigma}_m(\hat{c},\hat{d})$ denote the estimated residual variance-covariance matrices from the linear VECM and the m-regime TVECM, respectively. As mentioned in Hansen (1997), the distribution of the sup-LR statistic will be non-standard. A bootstrap procedure can be used to compute p-values for this test.

3.2.4. Hansen's procedures for the model specification test

The approach recently reviewed by Hansen (1999) uses a sequential testing procedure based on nested hypotheses. The current research applied Hansen's nested hypotheses tests based on unrestricted estimation of TAR models and TVECMs. This research started with a typical three-regime continuous symmetric threshold and symmetric adjustment BAND-TAR model for z_t as well as a three-regime symmetric threshold and symmetric adjustment BAND-TVECM for X_t. The symmetric BAND-TAR model is nested within an unrestricted TAR(3) model, while the symmetric BAND-TVECM is nested within an unrestricted TVECM(3). This nested structure enables a systematic specification analysis.

First consider was determining the number of regimes. Given that linearity is rejected in favor of threshold nonlinearity, in order to determine if a TAR(3) model for z_t is appropriate, Hansen (1999) was applied as well as a test of the null of a TAR(2) model against the alternative of a TAR(3) model using the F-statistic:

$$F_{2,3} = T\left(\frac{S_2 - S_3}{S_3}\right), \tag{3.22}$$

where S_2 and S_3 denote the sum of squared residuals from the estimation of an unrestricted TAR(2) model and an unrestricted TAR(3) model, respectively. Similarly, to determine if a TVECM(3) for X_t is appropriate one can test the null of a TVECM(2) against the alternative of a TVECM(3) using the LR statistic:

$$LR_{2,3} = T(\ln(|\hat{\Sigma}_2(\hat{c},\hat{d})|) - \ln(|\hat{\Sigma}_3(\hat{c},\hat{d})|)), \tag{3.23}$$

where $\hat{\Sigma}_2(\hat{c},\hat{d})$ and $\hat{\Sigma}_3(\hat{c},\hat{d})$ denote the estimated residual variance-covariance matrices from the unrestricted TVECM(2) and TVECM(3), respectively. As with the linearity tests discussed previously, the asymptotic distributions of $F_{2,3}$ and $LR_{2,3}$ are nonstandard and bootstrap methods can be used to compute approximate p-values.

3.3. Model 2: Hansen-Seo Two-Regime Threshold Cointegartion Model

3.3.1. Introduction

One pitfall of the Balke-Fomby (1997) procedure is that the authors do not offer a formal justification for their residual-based two-step approach. Hansen and Seo (2002) proposed a formal test procedure for threshold cointegration and they offered an algorithm to estimate model parameters. The Hansen-Seo model is a two-regime vector error correction model with only one cointegrating vector and with one built-in threshold effect in the error-correction term.

Based on a fully specified joint model, Hansen-Seo derived the maximum likelihood estimator of a threshold cointegration model. Under the null hypothesis of linearity, the threshold parameter is not identified, which causes a nuisance parameter problem. To get around this problem, the authors based their inference on a Sup-LM (Lagrange Multiplier) test statistic. They derived the asymptotic null distribution for this test statistic and discussed a bootstrap approximation to the sampling distribution. Two types of bootstrap algorithms are provided to approximate the sampling distribution: one is the fixed regressor bootstrap and the other one is a residual-based bootstrap. This section focuses on a bivariate threshold cointegration analysis based on the Hansen-Seo setting to analyze the dynamic relationships between corporate bond and Treasury rates.

3.3.2. A two-regime threshold cointegration model

Let x_t be a $p \times 1$ I(1) time series with one $p \times 1$ cointegrating vector β. Let $w_t(\beta) = \beta' x_t$ denote the I(0) error-correction term. Then, a linear vector error correction model (VECM) of order (L+1) may be expressed as:

$$\Delta x_t = A' X_{t-1}(\beta) + u_t, \tag{3.24}$$

where $X_{t-1}'(\beta) = [1, w_{t-1}(\beta), \Delta x_{t-1}, \Delta x_{t-2}, \ldots, \Delta x_{t-L}]$, with the following dimensions: $X_{t-1}(\beta)$ is $k \times 1$, where $k = p \times L + 2$, and A is $k \times p$. The error term u_t is a $p \times 1$ Martingale difference sequence with finite variance-covariance matrix $\Sigma = E(u_t u_t')$ of dimension $p \times p$. The Hansen-Seo approach is used to estimate the parameters (β, A, Σ) by maximum likelihood estimation given the assumption that the error terms u_t's are i.i.d. Gaussian distributed.[5]

A two-regime threshold cointegration model may be expressed as:

[5] Hereafter, it will be assumed that one element of a cointegrating vector is 1 and the other is ($-\beta$), and the error correction term will have the expression ($x_{1t} - \beta x_{2t}$).

$$\Delta x_t = \begin{cases} A_1' X_{t-1}(\beta) + u_t, & \text{if} \quad w_{t-1}(\beta) \le \gamma, \\ A_2' X_{t-1}(\beta) + u_t, & \text{if} \quad w_{t-1}(\beta) > \gamma, \end{cases}$$

where γ is the threshold parameter. This equation is rewritten as:

$$\Delta x_t = A_1' X_{t-1}(\beta) d_{1t}(\beta,\gamma) + A_2' X_{t-1}(\beta) d_{2t}(\beta,\gamma) + u_t, \tag{3.25}$$

where: $d_{1t}(\beta,\gamma) = I(w_{t-1}(\beta) \le \gamma)$, $d_{2t}(\beta,\gamma) = I(w_{t-1}(\beta) > \gamma)$, and $I(\cdot)$ is the Heaviside indicator function. To ensure the nonlinearity, Hansen-Seo, among others, suggested imposing the boundary constraint:

$$\pi_0 \le \Pr(w_{t-1} \le \gamma) \le 1 - \pi_0, \tag{3.26}$$

where $\pi_0 > 0$. Typically, the setting is $0.05 \le \pi_0 \le 0.15$. Since the u_t's are i.i.d. Gaussian, the likelihood function is:

$$L_n(A_1, A_2, \Sigma, \beta, \gamma) = -\frac{n}{2}\log|\Sigma| - \frac{1}{2}\sum_{t=1}^{n} u_t(A_1, A_2, \beta, \gamma)' \Sigma^{-1} u_t(A_1, A_2, \beta, \gamma),$$

where: $u_t(A_1, A_2, \beta, \gamma) = \Delta x_t - A_1' X_{t-1}(\beta) d_{1t}(\beta,\ \gamma) - A_2' X_{t-1}(\beta) d_{2t}(\beta,\ \gamma)$. The maximum likelihood estimators (MLEs) $(\hat{A}_1, \hat{A}_2, \hat{\Sigma}, \hat{\beta}, \hat{\gamma})$ are the values that maximize the likelihood function $L_n(A_1, A_2, \Sigma, \beta, \gamma)$.

3.3.3. Estimation procedure

Step 1: First, concentrate out (A_1, A_2, Σ) by holding (β, γ) fixed and compute the constrained MLE for (A_1, A_2, Σ). This is done through OLS estimation. Given Gaussian error terms, the maximum likelihood estimators are the same as ordinary least squared estimators:

$$\hat{A}_1(\beta,\gamma)=\left(\sum_{t=1}^{n}X_{t-1}(\beta)X_{t-1}(\beta)'d_{1t}(\beta,\gamma)\right)^{-1}\left(\sum_{t=1}^{n}X_{t-1}(\beta)\Delta x_t d_{1t}(\beta,\gamma)\right), \tag{3.27}$$

$$\hat{A}_2(\beta,\gamma)=\left(\sum_{t=1}^{n}X_{t-1}(\beta)X_{t-1}(\beta)'d_{2t}(\beta,\gamma)\right)^{-1}\left(\sum_{t=1}^{n}X_{t-1}(\beta)\Delta x_t d_{2t}(\beta,\gamma)\right), \tag{3.28}$$

$$\hat{u}_t(\beta,\gamma)=u_t(\hat{A}_1(\beta,\gamma),\,\hat{A}_2(\beta,\gamma),\beta,\gamma)\text{, and }\hat{\Sigma}(\beta,\gamma)=\frac{1}{n}\sum_{t=1}^{n}\hat{u}_t(\beta,\gamma)\hat{u}_t(\beta,\gamma)'. \tag{3.29}$$

Then, the concentrated likelihood function is:

$$L_n(\beta,\gamma)=L_n(\hat{A}_1(\beta,\gamma),\,\hat{A}_2(\beta,\gamma),\,\hat{\Sigma}(\beta,\gamma),\,\beta,\,\gamma)=-\frac{n}{2}\log\left|\hat{\Sigma}(\beta,\gamma)\right|-\frac{np}{2}. \tag{3.30}$$

Step 2: Compute the vector of parameters: $(\hat{\beta},\hat{\gamma})$. The MLE $\hat{u}_t=\hat{u}_t(\hat{\beta},\hat{\gamma})$ are the minimizers of $\log\left|\hat{\Sigma}(\beta,\gamma)\right|$ and the boundary constraint: $\pi_0\le n^{-1}\sum_{t=1}^{n}I(\beta'x_t\le\gamma)\le 1-\pi_0$. The MLE for A_1 and A_2 are thus $\hat{A}_1=\hat{A}_1(\hat{\beta},\hat{\gamma})$ and $\hat{A}_2=\hat{A}_2(\hat{\beta},\hat{\gamma})$.

3.3.4. Application to the term structure of interest rates

Let x_{1t} be the long rate and x_{2t} be the short rate. Then, a linear cointegrating VAR model is:

$$\begin{pmatrix}\Delta x_{1t}\\ \Delta x_{2t}\end{pmatrix}=\begin{pmatrix}\mu_1\\ \mu_2\end{pmatrix}+\begin{pmatrix}\alpha_1\\ \alpha_2\end{pmatrix}(x_{1t-1}-\beta x_{2t-1})+\begin{pmatrix}\Gamma_{11} & \Gamma_{12}\\ \Gamma_{21} & \Gamma_{22}\end{pmatrix}\begin{pmatrix}\Delta x_{1t-1}\\ \Delta x_{2t-1}\end{pmatrix}+\begin{pmatrix}u_{1t}\\ u_{2t}\end{pmatrix} \tag{3.31}$$

with $w_{t-1}=x_{1t-1}-\beta x_{2t-1}$. Here, if one sets $\beta = 1$ then the error-correction term becomes the interest rate spread. A two-regime model H_1 will enable all coefficients to differ depending upon $x_{1t-1}-\beta x_{2t-1}\le\gamma$ or $x_{1t-1}-\beta x_{2t-1}>\gamma$:

$$\begin{pmatrix}\Delta x_{1t}\\ \Delta x_{2t}\end{pmatrix}=\begin{pmatrix}\mu_1^{(1)}\\ \mu_2^{(1)}\end{pmatrix}+\begin{pmatrix}\alpha_1^{(1)}\\ \alpha_2^{(1)}\end{pmatrix}(x_{1t-1}-\beta x_{2t-1})+\begin{pmatrix}\Gamma_{11}^{(1)} & \Gamma_{12}^{(1)}\\ \Gamma_{21}^{(1)} & \Gamma_{22}^{(1)}\end{pmatrix}\begin{pmatrix}\Delta x_{1t-1}\\ \Delta x_{2t-1}\end{pmatrix}+\begin{pmatrix}u_{1t}\\ u_{2t}\end{pmatrix}\text{, if } x_{1t-1}-\beta x_{2t-1}\le\gamma,$$

$$\begin{pmatrix}\Delta x_{1t}\\ \Delta x_{2t}\end{pmatrix}=\begin{pmatrix}\mu_1^{(2)}\\ \mu_2^{(2)}\end{pmatrix}+\begin{pmatrix}\alpha_1^{(2)}\\ \alpha_2^{(2)}\end{pmatrix}(x_{1t-1}-\beta x_{2t-1})+\begin{pmatrix}\Gamma_{11}^{(2)} & \Gamma_{12}^{(2)}\\ \Gamma_{21}^{(2)} & \Gamma_{22}^{(2)}\end{pmatrix}\begin{pmatrix}\Delta x_{1t-1}\\ \Delta x_{2t-1}\end{pmatrix}+\begin{pmatrix}u_{1t}\\ u_{2t}\end{pmatrix},\text{ if } x_{1t-1}-\beta x_{2t-1}>\gamma .$$

3.4. Model 3: Enders-Siklos Threshold Cointegration Model

3.4.1. Introduction

Standard unit-root and cointegration tests, with their corresponding error correction representation, may entail a misspecification error, if the adjustment process is asymmetric. Two types of asymmetric tests in the form of threshold autoregressive (TAR) and momentum threshold autoregressive (M-TAR) adjustments representations were offered by Enders and Granger (1998), and Enders and Siklos (2001).

3.4.2. Review of Engle-Granger cointegration test and error correction representation

Conventional models often assume linearity and symmetric adjustment process for cointegrated variables. For example, consider the Engle and Granger (1987) two-step cointegration test. The first step applies the ordinary least squares method (OLS) to estimate the regression model:

$$x_{1t}=\beta_0+\beta_2 x_{2t}+\beta_3 x_{3t}+\ldots+\beta_n x_{nt}+\mu_t, \tag{3.32}$$

where x_{it} are individual I(1) processes, β_i's are the parameters, with i = 0, 2, …, n, and μ_t is a stochastic disturbance term that may be serially correlated.

The second step uses a Dickey-Fuller (1979, 1981) type of unit root test applied to the OLS estimate of ρ in the following regression:

$$\Delta\hat{\mu}_{t-1} = \rho\hat{\mu}_{t-1} + \varepsilon_t, \tag{3.33}$$

where $\hat{\mu}_t$ is the residual from the OLS estimate of (3.32) and ε_t is a white noise process.

In the Engle-Granger cointegration test, the null hypothesis of no cointegration is H_0: $\rho = 0$. Engle and Granger showed that rejecting the null hypothesis of no cointegration (i.e., accepting the alternative hypothesis of H_A: $-2 < \rho < 0$) implies that the error process in (3.33) is stationary with mean zero. This also implies the entire system of x_{1t}, x_{2t}, …, and x_{nt} are cointegrated with a symmetric adjustment mechanism towards the long run equilibrium (or the attractor) β_0.

The Granger Representation Theorem suggests that if $\rho \neq 0$ (i.e., the system of x_{1t}, x_{2t}, …, and x_{nt} are cointegrated) then (3.32) and (3.33) will guarantee the existence of an error-correction representation in the form of:

$$\Delta x_{1t} = \alpha_1(x_{1t-1} - \beta_0 - \beta_2 x_{2t-1} - \beta_3 x_{3t-1} - \ldots - \beta_n x_{nt-1}) + \varepsilon_{1t}. \tag{3.34}$$

Similar representations can be derived for x_{2t}, x_{3t}, …, and x_{nt}.

Enders and Siklos (2001) pointed out that if the adjustment process is, indeed, asymmetric then the Engle-Granger cointegration test is misspecified and the error-correction mechanism is unable to capture the actual adjustment process. They suggested an alternative specification of the error-correction model in the form of threshold autoregressive framework.

3.4.3. Enders-Siklos cointegration test

Enders and Siklos (2001) developed asymmetric cointegration tests by incorporating threshold autoregressive (TAR) and momentum threshold autoregressive (M-TAR) adjustments into the unit-root tests of the residuals of the cointegration regression, such as equation (3.32). Assuming the deviations from long run equilibrium behave as a TAR process:

$$\Delta\mu_t = I_t\rho_1\mu_{t-1} + (1 - I_t)\rho_2\mu_{t-1} + \varepsilon_t, \tag{3.35}$$

where I_t is the Heaviside indicator function, such that:

$$I_t = \begin{cases} 1, & \text{if } \mu_{t-1} \geq 0 \\ 0, & \text{if } \mu_{t-1} < 0. \end{cases} \tag{3.36}$$

In the M-TAR model, the Heaviside indicator function M_t is defined as:

$$M_t = \begin{cases} 1, & \text{if } \Delta\mu_{t-1} \geq 0 \\ 0, & \text{if } \Delta\mu_{t-1} < 0. \end{cases} \tag{3.37}$$

In general, the asymmetric adjustment coefficients of ρ_1 and ρ_2 allow a state-dependent autoregressive decay process. For example, in the M-TAR model: if $\Delta\mu_{t-1} \geq 0$, the adjustment is $\rho_1\mu_{t-1}$; while if $\Delta\mu_{t-1} < 0$ then the adjustment is $\rho_2\mu_{t-1}$. Consider equations (3.35) and (3.36), the TAR model. If $|\rho_2| > |\rho_1|$, say $\rho_1 = -0.2$, $\rho_2 = -0.8$ then positive deviations from the long-run cointegration equilibrium are more persistent than negative deviations. In other words, there is a slow adjustment when the equilibrium error is above the attractor, while there is an accelerated adjustment when the equilibrium error is below the attractor. This adjustment mechanism captures the feature of "deep" cyclical processes documented by Sichel (1993).

Consider equations (3.35) and (3.37), the M-TAR model, which allow the autoregressive decay to depend on $\Delta\mu_{t-1}$. Again, if $|\rho_2| > |\rho_1|$, say, $\rho_1 = -0.2$, $\rho_2 = -0.8$ then there is little decay when $\Delta\mu_{t-1}$ is positive, but substantial decay when $\Delta\mu_{t-1}$ is negative; in such a situation, increases tend to persist, but decreases tend to revert quickly toward the attractor. Hence, the M-TAR model could easily capture the "sharp" movements documented in DeLong and Summer (1986) and Sichel (1993).

Enders and Siklos (2001) offered some extensions to modify the basic threshold cointegration model given previously. The first modification is to allow a non-zero drift term as the linear attractor, which can be expressed as:

$$\Delta\mu_t = I_t\rho_1(\mu_{t-1} - a_0) + (1 - I_t)\rho_2(\mu_{t-1} - a_0) + \varepsilon_t, \tag{3.38}$$

where I_t is the Heaviside indicator function, such that:

$$I_t = \begin{cases} 1, & \text{if } \mu_{t-1} \geq a_0 \\ 0, & \text{if } \mu_{t-1} < a_0. \end{cases} \tag{3.39}$$

The second modification involves a drift and linear trend as attractor with the expression:

$$\Delta\mu_t = I_t\rho_1[\mu_{t-1} - a_0 - a_1(t-1)] + (1 - I_t)\rho_2[\mu_{t-1} - a_0 - a_1(t-1)] + \varepsilon_t, \tag{3.40}$$

where I_t is the Heaviside indicator function, such that:

$$I_t = \begin{cases} 1, & \text{if } \mu_{t-1} \geq a_0 + a_1(t-1) \\ 0, & \text{if } \mu_{t-1} < a_0 + a_1(t-1). \end{cases} \tag{3.41}$$

The third modification involves higher-order terms of the error process to purge possible auto-correlation:

$$\Delta\mu_t = I_t\rho_1\mu_{t-1} + (1 - I_t)\rho_2\mu_{t-1} + \sum_{i=1}^{p-1}\gamma_i\Delta\mu_{t-i} + \varepsilon_t. \tag{3.42}$$

To ensure the stationarity of μ_t, all roots of the characteristic equation of $(1 - \gamma_1 r - \gamma_2 r^2 - \ldots - \gamma_{p-1} r^{p-1}) = 0$ must lie outside the unit circle. More complex model can be built on combinations of the above modifications, e.g., an M-TAR model with a non-zero attractor with p-th order process can be written as:

$$\Delta\mu_t = M_t\rho_1\mu_{t-1} + (1 - M_t)\rho_2\mu_{t-1} + \sum_{i=1}^{p-1}\gamma_i\Delta\mu_{t-i} + \varepsilon_t, \tag{3.43}$$

M_t is the Heaviside indicator function and a_0 is the linear attractor, such that:

$$M_t = \begin{cases} 1, & \text{if } \Delta\mu_{t-1} \geq a_0, \\ 0, & \text{if } \Delta\mu_{t-1} < a_0. \end{cases} \tag{3.44}$$

3.4.4. Chan's consistent estimator of the threshold

Tsay (1989) and Chan (1993) offered methodologies for model building if the underlying variables are within the threshold autoregressive framework. Tong (1983) also demonstrated that if the adjustment process is asymmetric then the sample mean is a biased estimator of the attractor. To rectify this bias, Chan (1993) showed that searching over all values of a_0, so as to minimize the sum of squared errors from the fitted model, yields a super-consistent estimator of the threshold.

3.4.5. Estimation procedures

The focus of this section is on applying Enders-Siklos cointegration tests in conjunction with Chan's consistent estimate of threshold. The following outlines the procedures:

Case 1: τ equals 0

Step1: Regress one of the variables on a constant and the other variable(s) and save the residuals sequence $\{\hat{\mu}_t\}$. Next, set the Heaviside indicator function according to (3.36) or (3.37) using $\tau = 0$. Estimate a regression equation in the form of (3.35) and record the larger of the t statistics for the null hypothesis of $\rho_i = 0$ along with the F statistic for the null hypothesis H_0: $\rho_1 = \rho_2 = 0$. Compare the F-statistic with appropriate critical values simulated by Enders and Siklos (2001) in Tables 1 or 2.

Step 2: If the alternative hypothesis of stationarity is accepted, it is possible to test for symmetric adjustment (i.e., $\rho_1 = \rho_2$). When the value of threshold is known, Enders and Falk (1999) state that bootstrap t intervals and classic t intervals work well enough to be recommended in practice.

Step3: Diagnostic checking of the residuals should be undertaken to ascertain whether the $\hat{\varepsilon}_t$ series could reasonably be characterized by a white-noise process. If the residuals are serially correlated, return to Step 2 and reestimate the model in the form:

$$\Delta\hat{\mu}_t = I_t\rho_1\hat{\mu}_{t-1} + (1 - I_t)\rho_2\hat{\mu}_{t-1} + \gamma_1\Delta\hat{\mu}_{t-1} + \cdots + \gamma_p\Delta\hat{\mu}_{t-p} + \varepsilon_t$$

for the TAR model. For the M-TAR case, replace I_t with M_t as specified in (3.37). Lag lengths can be determined by an analysis of the regression residuals and/or using model-selection criteria such as AIC/BIC.

Case 2: τ is unknown

Step 1: Regress one of the variables on a constant and the other variable(s) and save the residuals sequence $\{\hat{\mu}_t\}$.

Step 2: For TAR case, the estimated residual series is sorted in ascending order and called $\mu_1^\tau < \mu_2^\tau < \cdots < \mu_T^\tau$, where T denotes the number of usable observations. Discard the largest and smallest 15% of the $\{\mu_i^\tau\}$ values. Each of the remaining 70% of the values is considered as possible thresholds.

Step 3: For each of these possible thresholds, estimate an equation in the form of (3.35) and (3.36). The estimated threshold yielding the lowest residual sum of squares is deemed to be the appropriate estimate of the threshold. For the M-TAR case, the potential thresholds are $\Delta\mu_1^\tau < \Delta\mu_2^\tau < \cdots < \Delta\mu_T^\tau$. For each of these possible thresholds, estimate an equation in the form of (3.35) and (3.37). The estimate of the threshold is the estimated threshold yielding the lowest residual sum of squares.

Step 4: Reestimate the model by incorporating the estimated threshold.

Step 5: Inference concerning the individual values of ρ_1 and ρ_2 and the restriction that $\rho_1 = \rho_2$ is problematic when the true value of the threshold τ is unknown. The property of asymptotic multivariate normality has not been established for this case. Chan and Tong (1989) conjectured that utilizing a constant estimate should establish the asymptotic normality of the coefficients. Enders and Falk (1999) found that the inversion of the bootstrap distribution for the likelihood ratio statistic provides reasonably good coverage in small samples.

CHAPTER 4. DATA

4.1. Introduction

This research used monthly averages of daily rates for 10-year constant maturity Treasury Note, the Ibbotson Bond Index for 20-year Treasury bonds, and Moody's Aaa and Baa seasoned bond indices. The data covered the period from January 1960 to December 1997, providing a total of 456 observations. These series were selected because of their long history and because it would be easier to compare the results with Neal, Rolph and Morris (2000, hereafter NRM). The Ibbotson 20-year Treasury Index was incorporated, since the 30-year constant maturity index does not start until 1977 and the 20-year constant maturity index is unavailable between 1987 and 1992. Thus, the Ibbotson Index was used as a proxy of long-term (20 years) Treasury bond rates.

The Moody indices are constructed from an equally weighted sample of yields on 75 to 100 bonds issued by non-financial corporations. As with other corporate bond indices, the Moody indices have some factors that will cause bias. One is the downward bias due to the call feature of bonds (Aaa or Baa) included in the indices. Duffee (1998) suggested that this embedded option (call feature) would cause a negative relationship between credit spreads and non-callable Treasury yields, since a decline in the yields will increase the value of the option.

The second factor that will also cause downward bias is from the construction of the corporate bond indices. Usually, the indices are constructed at the end of the month with bonds rating Aaa or Baa. If some Baa bonds are downgraded before the end of the month then other Baa bonds will replace them. Excluding these downgraded Baa bonds will cause the index to be understated, because the downgraded bonds usually have a greater rise in yield than bonds with unchanged ratings. Other factors include tax differentials and trends in the credit quality of the indices. The magnitude of this bias is difficult to assess. NRM offers a thorough discussion of the pros and cons of the Moody indices and interested readers should see their paper for details.

Table 4.1 provides summary statistics of Moody's Aaa, and Baa corporate bond rates, the 10-year Treasury Index (Tsy), and the Ibbotson Index (Ibb). Notice that the means of these series are in order from lowest to highest: Tsy, Ibb, Aaa, and Baa. Table 4.1 also presents summary statistics for the spreads between the Moody's corporate bond series (Aaa and Baa) and the Treasury rates (Tsy and Ibb).

Table 4.2 calculates the autocorrelations for Tsy, Ibb, Aaa, and Baa. The high degree of persistence is likely signaling the presence of a unit root or near unit root process. NRM (2000) reports that the levels of the interest rates appear non-stationary while the changes appear stationary based upon the results of Dickey-Fuller and Phillips-Perron unit root tests. These results confirm the conclusions of quite a few studies on the presence of unit roots in nominal interest rates. For example, see Stock and Watson (1988), Hall, Anderson and Granger (1992), and Enders and Granger (1998) for short-term rates. See Campbell and Shiller (1987) and Mehra (1994) for long-term rates.

The Dickey-Fuller and Phillips-Perron unit root tests test the null of a unit root against the alternative that the process is a stationary linear process. The next section reports the Enders and Granger (1998) unit root test results, which are appropriate when the alternative is a stationary nonlinear process.

4.2. Review of the Enders-Granger Unit-Root Test

The first-order threshold autoregressive (TAR) representation for the zero-mean time series $\{y_t\}$, which allows for asymmetric adjustment to the long run equilibrium, is formulated as:

$$\Delta y_t = \begin{cases} \rho_1 y_{t-1} + \varepsilon_t, & \text{if } y_{t-1} \geq 0, \\ \rho_2 y_{t-1} + \varepsilon_t, & \text{if } y_{t-1} < 0, \end{cases} \tag{4.1}$$

where ε_t is a white noise process.

A sufficient condition for the stationarity of $\{y_t\}$ is $-2 < (\rho_1, \rho_2) < 0$. If $\rho_1 = \rho_2 = 0$ then $\{y_t\}$ is a standard random walk. One can rewrite the above asymmetric adjustment process as:

$$\Delta y_t = I_t \rho_1 y_{t-1} + (1 - I_t)\rho_2 y_{t-1} + \varepsilon_t, \tag{4.2}$$

where I_t is the Heaviside indicator function, such that:

$$I_t = \begin{cases} 1, & \text{if } y_{t-1} \geq 0, \\ 0, & \text{if } y_{t-1} < 0. \end{cases} \tag{4.3}$$

The momentum-threshold autoregressive (M-TAR) representation is the same as the TAR representation, except the Heaviside indicator function is defined as:

$$I_t = \begin{cases} 1, & \text{if } \Delta y_{t-1} \geq 0, \\ 0, & \text{if } \Delta y_{t-1} < 0. \end{cases} \tag{4.4}$$

However, most of the time series $\{y_t\}$ has a non-zero mean a_0, then above TAR model should be modified as:

$$\Delta y_t = I_t \rho_1 (y_{t-1} - a_0) + (1 - I_t)\rho_2 (y_{t-1} - a_0) + \varepsilon_t, \tag{4.2'}$$

where I_t is the Heaviside indicator function, such that:

$$I_t = \begin{cases} 1, & \text{if } y_{t-1} \geq a_0, \\ 0, & \text{if } y_{t-1} < a_0. \end{cases} \tag{4.3'}$$

Similarly, for the M-TAR model, the Heaviside indicator function should be:

$$I_t = \begin{cases} 1, & \text{if } \Delta y_{t-1} \geq a_0, \\ 0, & \text{if } \Delta y_{t-1} < a_0. \end{cases} \tag{4.4'}$$

The next section briefly describes the Enders and Granger (1998) procedure for testing the unit root null, $\rho_1 = \rho_2 = 0$ against threshold autoregressive (TAR) and momentum threshold autoregressive (M-TAR) adjustment alternatives under two different types of attractor settings.

4.3. Test procedure

The attractor $y_{t-1} = 0$ (or $\Delta y_{t-1} = 0$) in equation (4.3) (or (4.4)) is a special case of the more general attractors $y_{t-1} = a_0$ (a non-zero constant attractor) or $y_{t-1} = a_0 + a_1(t-1)$ (a trend attractor). Enders and Granger (1998) pointed out that in the more general case, if the attractor is known, the data could be transformed so that the attractor $y_{t-1} = 0$ ($\Delta y_{t-1} = 0$) is applicable. Otherwise, one needs to estimate the values of a_0 and a_1 from the data. For the current study, visual investigation of the spreads of indices (Aaa - Tsy), (Aaa - Ibb), (Baa - Tsy) and (Baa - Ibb) suggested that the spreads of indices all have non-zero sample means but do not show any time trend pattern. Hence, the asymmetric unit root test for TAR and M-TAR adjustments are carried out assuming all the spread series have a non-zero constant attractor.

The test procedure involves the following sequence of steps:

Step 1: De-mean the spreads by removing the sample mean from each interest rate differential series, yielding the residual series $\{\hat{y}_t\}$.

Step 2: Apply OLS to estimate the following regression equation:

$$\Delta\hat{y}_t = I_t\rho_1\hat{y}_{t-1} + (1 - I_t)\rho_2\hat{y}_{t-1} + \hat{\varepsilon}_t. \tag{4.5}$$

Because $\{\hat{y}_t\}$ is the "demeaned" series, equation (4.5) is equivalent to

$\Delta y_t = I_t\rho_1(y_{t-1} - \hat{a}_0) + (1 - I_t)\rho_2(y_{t-1} - \hat{a}_0) + \hat{\varepsilon}_t$, where $\hat{a}_0$ is the estimated sample mean of the $\{y_t\}$ sequence. The indicator function I_t will be determined by the type of asymmetry under consideration (i.e., TAR or M-TAR specification). Diagnostic checking of the regression residuals ($\hat{\varepsilon}_t$), for the purging of autocorrelation and the AIC and SBC criteria are used to determine the lag lengths. If the errors in Equation (4.5) are serially correlated, it is possible to implement an augmented TAR (or M-TAR) specification for the residuals. Therefore, equation (4.5) is replaced by:

$$\Delta\hat{y}_t = I_t\rho_1\hat{y}_{t-1} + (1 - I_t)\rho_2\hat{y}_{t-1} + \sum_{i=1}^{p}\alpha_i\Delta\hat{y}_{t-i} + \hat{\varepsilon}_t. \tag{4.5'}$$

Step 3: The test statistics for the null hypothesis of nonstationarity: $\rho_1 = \rho_2 = 0$, are compared with the appropriate critical values in Table 4.3 of Enders and Granger (1998).

The distribution of the Enders-Granger unit root test statistic depends on the sample size and the presence of various deterministic regressors in the attractor: (A) zero attractor, (B) non-zero constant attractor, or (C) linear trend attractor. The corresponding statistics are the Φ statistic, the Φ_μ statistic, and the Φ_T statistic for TAR specification (and the Φ^*, the Φ^*_μ, and the Φ^*_T for M-TAR specification.) If the unit root null hypothesis is rejected, the restriction of symmetric adjustment ($\rho_1 = \rho_2$) versus the alternative of asymmetric adjustment can be tested using standard F-statistics, since the least squares estimates of ρ_1 and ρ_2 converge to multivariate normal distributions, if the $\{\hat{y}_t\}$ sequence is stationary (Tong 1983). However, Enders and Falk (1998) and Hansen (1997) showed that small sample properties of the OLS estimates of ρ_1 and ρ_2 have inflated standard errors and the OLS estimates may have poor convergent properties. As such, inference concerning the individual values of ρ_1 and ρ_2 is problematic. Hence, it is not appropriate to implement a simple t-test.

Step 4: Tong (1983) demonstrated that, if the adjustment process is asymmetric, the sample mean is a biased estimate of the attractor. To rectify this bias, Chan (1993) showed that selecting a_0 to minimize the sum of squared errors from the fitted model yields a super-consistent estimate of the threshold. Since the spread of indices all have non-zero means and one has no prior information about the true value of the attractor, it is appropriate to apply this method in conjunction with the threshold (and momentum threshold) autoregressive estimation. Referring to the resulting model as the consistent threshold autoregressive (C-TAR and momentum-consistent threshold autoregressive, M-C TAR) model, to distinguish it from TAR (and M-TAR) model, fit with the biased estimate of the attractor. In the C-TAR model, the Heaviside indicator function has the form:

$$I_t = \begin{cases} 1, & \text{if } y_{t-1} \geq a_0, \\ 0, & \text{if } y_{t-1} < a_0. \end{cases} \tag{4.6}$$

While in this M-C TAR model, the Heaviside indicator function has the form:

$$I_t = \begin{cases} 1, & \text{if } \Delta y_{t-1} \geq a_0, \\ 0, & \text{if } \Delta y_{t-1} < a_0, \end{cases} \tag{4.6'}$$

where a_0 is the nonzero constant attractor.

4.4. Results

The following results are based on the data analysis:

1. This study focused on four pairs of yield spreads: (Aaa, Tsy), (Aaa, Ibb), (Baa, Tsy), and (Baa, Ibb). Tables 4.4 to 4.7 report five types of unit root tests: Dickey-Fuller Test, Threshold Autoregressive Test (TAR), Momentum Threshold Autoregressive Test (M-TAR), Consistent Threshold Autoregressive Test (C-TAR) and Momentum-Consistent Autoregressive Test (M-C TAR).
2. At the 5% significance level, with sample size of 500, the critical value is -2.87 for the Dickey-Fuller τ_{μ} test. Entries, in the brackets of the third row-the third column of Tables 4.4 to 4.7, report the t-statistics for the Dickey-Fuller tests for the current study: -3.462 (-6.505, -2.958 and -3.126) for the yield spread of (Aaa, Tsy) ((Aaa, Ibb), (Baa, Tsy), and (Baa, Ibb)). Since all test statistics are greater than the critical value (in the absolute value sense), it is possible to reject the null of a unit root process. That is, the evidence indicates that the yield spreads are stationary for all four pairs. This finding is consistent with the work of Enders and Granger (1998). However, the motivation for the Enders-Granger study was that they were concerned that failure to reject the unit root null could result from the lack of power against threshold stationary alternatives. In the current study, the unit root null is rejected. Moreover, the estimate of the adjustment parameter of ρ (the parameter estimate reported in the second row-the third column) lies between the estimate of ρ_1 and ρ_2 (reported at the third and fourth columns, the fourth, sixth, eighth, and tenth row) as expected.

3. The residual sequence obtained from the first step of the procedure is to fit the TAR and M-TAR models using the threshold $a_0 = 0$. The AIC and BIC diagnostic statistics select two lags for the yield spread of the (Aaa, Tsy) and (Baa, Tsy) pairs, one lag for the yield spread of the (Aaa, Ibb) pair, and three lags for the yield spread of the (Baa, Ibb) pair. For each spread, the optimal lag lengths are consistent across all four models.
4. The statistic Φ, in the sixth column of Tables 4.4 to 4.7 (under the column heading Φ), reports the test of the unit root null hypothesis: $\rho_1 = \rho_2 = 0$. At the conventional 5% significance level and at sample size of 500, the hypothesis $\rho_1 = \rho_2 = 0$ is rejected for all four pairs of yield spreads. This conclusion is drawn by comparing test statistics to the appropriate critical values of Φ_μ and Φ_μ^* (4.56 and 4.95, corresponding to TAR and M-TAR, respectively). For example, the test statistic Φ is 6.105 (7.621) for the TAR (M-TAR) specification of yield spread pair (Aaa, Tsy). Note that the only exception is the M-TAR specification for the (Baa, Ibb) pair. In addition, notice that these results are consistent with the results from the Dickey-Fuller tests (as stated in result #2).
5. Given the conclusion of the stationarity of the yield spreads, the standard F-tests of the null hypothesis of symmetric adjustment: $\rho_1 = \rho_2$ are conducted. The seventh columns of Tables 4.4 to 4.7 report the test results. In general, under TAR and M-TAR specifications, the null hypothesis of $\rho_1 = \rho_2$ was not rejected, with only two exceptions. The test results rejected the symmetry hypothesis for the yield spread

pairs of (Aaa, Tsy) and (Baa, Tsy), under the M-TAR specification, and hence concluded asymmetric adjustments in the process of yield spreads, at the 10% significance level. Under the C-TAR specification, the test results rejected the null of symmetry at 10% significance level for the pairs of (Aaa, Tsy) and ((Baa, Tsy). Furthermore, under the M-C TAR specification, the test results rejected the null of symmetry: $\rho_1 = \rho_2$ at conventional 5% significance level with only one exception of (Baa, Ibb) yield spread pair.

6. As measured by AIC and BIC, the selected asymmetric model, which incorporates the consistent threshold estimator, fits the data marginally better than all other models. Overall, the asymmetric specification yields a smaller AIC and BIC than the symmetric model.
7. The results indicate that the yield spreads are stationary and the adjustment mechanisms are asymmetric for the M-C TAR model. However, the TAR and M-TAR models do not support asymmetry. In general, TAR specification can capture aspects of "deepness", i.e., $-1 < \rho_1 < \rho_2 < 0$, the negative phase will tend to be more persistent than the positive phase. Three pairs of yield spreads with TAR specifications capture this characteristic except the pair of (Aaa, Tsy). Furthermore, the estimates infer four (three) yield spreads M-C TAR (M-TAR) processes capture the sharpness of the series, in which the autoregressive decay is relatively sharp, when it is decreasing rather than when it is increasing (i.e., $|\rho_1| < |\rho_2|$).

8. Last, the M-C TAR model offers evidence of asymmetric adjustments that is in agreement with the findings of nonlinearities and mean-reversion of Jones (1999), for same sample period.

The empirical results are reported and discussed in Chapter 5.

Table 4.1. Summary statistics

	Mean	Std Error	Minimum	Maximum
Aaa	8.1418	2.6075	4.1900	15.4900
Baa	9.1475	2.9730	4.7800	17.1800
Tsy	7.4582	2.5751	3.7100	15.3200
Ibb	7.5808	2.5389	3.8031	14.8236
ΔAaa	0.0047	0.2425	-1.1800	1.2900
ΔBaa	0.0044	0.2188	-1.0200	1.1500
ΔTsy	0.0024	0.3079	-1.7600	1.6100
ΔIbb	0.0035	0.3218	-1.6403	1.1380
Aaa-Tsy	0.6836	0.3772	-0.1700	1.6000
Aaa-Ibb	0.5610	0.3511	-0.5273	2.0190
Baa-Tsy	1.6893	0.6440	0.2900	3.8200
Baa-Ibb	1.5666	0.6634	0.3544	4.1890

Note: The statistics are based on monthly data from 1960:1 to 1997:12. The Aaa and Baa series are from Moody's, the Tsy (10-year Treasury series) is a constant maturity series from the Board of Governors and Ibb is Ibbotson 20-year Treasury index.

Table 4.2. Sample autocorrelations

	Aaa		Baa		Tsy		Ibb	
Lag	Coeffi	Q-Stat	Coeffi	Q-Stat	Coeffi	Q-Stat	Coeffi	Q-Stat
1	0.996	455	0.997	457	0.993	452	0.992	452
2	0.988	904	0.992	909	0.981	895	0.983	896
3	0.981	1348	0.986	1357	0.971	1330	0.974	1333
4	0.975	1788	0.980	1801	0.961	1756	0.968	1766
5	0.969	2223	0.973	2240	0.952	2176	0.961	2194
6	0.962	2652	0.966	2673	0.941	2587	0.954	2616

Note: The statistics are based on monthly data from 1960:1 to 1997:12. The Aaa and Baa series are from Moody's, the 10-year Treasury series is a constant maturity series from the Board of Governors and the Ibb is from Ibbotson 20-year Treasury index. The Box-Ljung Q-Statistic tests the null hypothesis that the series is not serially correlated. This statistic is distributed $\chi(n)$, where n is the number of lags. The null hypothesis is rejected at a significance level of less than 0.1% for all lags.

Table 4.3. The critical values for rejecting the null hypothesis of a unit root

(A). No estimated deterministic components

	Φ statistic			Φ^* statistic		
Sample size	90%	95%	99%	90%	95%	99%
250	3.10	3.82	5.53	2.68	3.41	5.10
1,000	3.04	3.75	5.36	2.51	3.21	4.85

(Source of critical values: Enders and Granger (1998), Table 1, page 306.)

(B). Estimated constant attractor

	Φ_μ statistic			Φ_μ^* statistic		
Sample size	90%	95%	99%	90%	95%	99%
250	3.74	4.56	6.47	4.05	4.95	6.99
1,000	3.74	4.56	6.41	4.05	4.95	6.91

(C). Estimated trend attractor

	Φ_T statistic			Φ_T^* statistic		
Sample size	90%	95%	99%	90%	95%	99%
250	5.18	6.12	8.23	5.64	6.65	8.85
1,000	5.15	6.08	8.12	5.60	6.57	8.74

Table 4.4. Unit-Root tests for Aaa-Tsy (sample period = 1960:1 to 1997:12, n = 456)

Model	Lag	ρ_1	ρ_2	AIC/BIC[a]	Φ [b]		Q(4)[d]
Dickey-	2	-0.059		939/956			1.424
Fuller		(-3.462)[e]					(0.840)
TAR	2	-0.051	-0.067	937/954	6.105	0.227	1.386
$\hat{a}_0 = 0.6836$		(-2.232)	(-2.755)[f]			(0.634)	(0.847)
M-TAR	2	-0.025	-0.085	936/953	7.621	3.180	1.468
$\hat{a}_0 = 0.6836$		(-0.993)	(-3.799)			(0.075)	(0.832)
C-TAR	2	-0.037	-0.102	936/952	7.750	3.432	1.390
$\hat{a}_0 = 0.2600$		(-1.810)	(-3.548)			(0.065)	(0.846)
M-C TAR	2	-0.034	-0.135	932/949	9.505	6.851	1.807
$\hat{a}_0 = 0.5936$		(-1.786)	(-4.016)			(0.009)	(0.771)

Note: Aaa denotes the Moody's corporate bonds, and Tsy denotes the Treasury notes.
[a] AIC = T*ln(residual sum of squares) + 2*n; BIC = T*ln(residual sum of squares) + n*ln(T), where n = number of regressors and T = number of usable observations.
[b] Entries in this column are the sample F-statistics for testing the null of $\rho_1 = \rho_2 = 0$.
[c] Entries in this column are the sample F-statistics for the null hypothesis that adjustments are symmetric. The corresponding significance levels are contained in brackets.
[d] Q(4) is the Ljung-Box statistics for the joint hypotheses of no serial correlation among the first four residuals.
[e] Entries in the brackets of this column are the t-statistics for the null hypothesis $\rho_1 = 0$.
[f] Entries in the brackets of this column are the t-statistics for the null hypothesis $\rho_2 = 0$.

Table 4.5. Unit-Root tests for Aaa-Ibb (sample period = 1960:1 to 1997:12, n = 456)

Model	Lag	ρ_1	ρ_2	AIC/BIC[a]	Φ [b]	$\rho_1 = \rho_2$ [c]	Q(4)[d]
Dickey-	1	-0.228		1497/1510			5.152
Fulle		(-6.505)[e]					(0.272)
TAR	1	-0.233	-0.221	1494/1507	21.174	0.031	5.169
$\bar{a}_0 = 0.5608$		(-5.390)	(-4.034)[f]			(0.861)	(0.270)
M-TA	1	-0.237	-0.218	1494/1506	21.202	0.081	5.216
$\bar{a}_0 = 0.5608$		(-5.128)	(-4.285)			(0.776)	(0.266)
C-TAR	1	-0.241	-0.207	1497/1510	21.297	0.255	5.161
$\bar{a}_0 = 0.7000$		(-5.567)	(-3.817)			(0.614)	(0.271)
M-C TAR	1	-0.200	-0.408	1493/15	23.881	4.980	4.359
$\bar{a}_0 = 0.3508$		(-5.375)	(-4.648)			(0.026)	(0.360)

Note: Aaa denotes the Moody's corporate bonds, and IBB denotes the Ibbotson Treasury Index.

[a] AIC = T*ln(residual sum of squares) + 2*n; BIC = T*ln((residual sum of squares) + n*ln(T),
where n = number of regressors and T = number of usable observations.

[b] Entries in this column are the sample F-statistics for testing the null of $\rho_1 = \rho_2 = 0$.

[c] Entries in this column are the sample F-statistics for the null hypothesis that adjustments are symmetric. The corresponding significance levels are contained in brackets.

[d] Q(4) is the Ljung-Box statistics for the joint hypotheses of no serial correlation among the first four residuals.

[e] Entries in the brackets of this column are the t-statistics for the null hypothesis $\rho_1 = 0$.

[f] Entries in the brackets of this column are the t-statistics for the null hypothesis $\rho_2 = 0$.

Table 4.6. Unit-Root tests for Baa-Tsy (sample period = 1960:1 to 1997:12, n = 456)

Model	Lag	ρ_1	ρ_2	AIC/BIC[a]	Φ[b]	$\rho_1 = \rho_2$[c]	Q(4)[d]
Dickey-	2	-0.038		1193/1209			4.715
Fuller		(-2.958)[e]					(0.318)
TAR	2	-0.046	-0.029	1190/1206	4.602	0.452	4.716
$\hat{a}_0 = 1.6893$		(-2.658)	(-1.496)[f]			(0.502)	(0.318)
M-TAR	2	-0.006	-0.069	1187/1203	7.326	5.795	5.124
$\hat{a}_0 = 1.6893$		(-0.302)	(-3.810)			(0.016)	(0.275)
C-TAR	2	-0.062	-0.016	1190/1206	5.982	3.159	4.430
$\hat{a}_0 = 2.2200$		(-3.347)	(-0.902)			(0.076)	(0.351)
M-C TAR	2	0.000	-0.098	1179/1196	11.269	13.531	5.252
$\hat{a}_0 = 1.6393$		(0.025)	(-4.747)			(0.000)	(0.262)

Note: Baa denotes the Moody's corporate bonds, and Tsy denotes the Treasury notes.
[a] AIC = T*ln(residual sum of squares) + 2*n; BIC = T*ln((residual sum of squares) + n*ln(T),
where n = number of regressors and T = number of usable observations.
[b] Entries in this column are the sample F-statistics for testing the null of $\rho_1 = \rho_2 = 0$.
[c] Entries in this column are the sample F-statistics for the null hypothesis that adjustments are symmetric. The corresponding significance levels are contained in brackets.
[d] Q(4) is the Ljung-Box statistics for the joint hypotheses of no serial correlation among the first four residuals.
[e] Entries in the brackets of this column are the t-statistics for the null hypothesis $\rho_1 = 0$.
[f] Entries in the brackets of this column are the t-statistics for the null hypothesis $\rho_2 = 0$.

Table 4.7. Unit-Root tests for Baa-Ibb (sample period = 1960:1 to 1997:12, n = 456)

Model	Lag	ρ_1	ρ_2	AIC/BIC[a]	Φ [b]	$\rho_1 = \rho_2$ [c]	Q(4)[d]
Dickey-	3	-0.065		1617/1638			1.952
Fuller		(-3.126)[e]					(0.745)
TAR	3	-0.070	-0.057	1613/1634	4.935	0.101	1.938
$\hat{a}_0 = 1.5664$		(-2.698)	(-1.719)[f]			(0.751)	(0.747)
M-TAR	3	-0.060	-0.069	1617/1638	4.905	0.043	1.941
$\hat{a}_0 = 1.5664$		(-2.009)	(-2.386)			(0.836)	(0.747)
C-TAR	3	-0.086	-0.038	1616/1636	5.594	1.391	1.797
$\hat{a}_0 = 2.1800$		(-3.147)	(-1.243)			(0.239)	(0.773)
M-C TAR	3	-0.046	-0.102	1616/1636	5.697	1.591	1.856
$\hat{a}_0 = 1.5064$		(-1.796)	(-2.819)			(0.208)	(0.762)

Note: Baa denotes the Moody's corporate bonds, and IBB denotes the Ibbotson Treasury Index.

[a] AIC = T*ln(residual sum of squares) + 2*n; BIC = T*ln((residual sum of squares) + n*ln(T),
where n = number of regressors and T = number of usable observations.

[b] Entries in this column are the sample F-statistics for testing the null of $\rho_1 = \rho_2 = 0$.

[c] Entries in this column are the sample F-statistics for the null hypothesis that adjustments are symmetric. The corresponding significance levels are contained in brackets.

[d] Q(4) is the Ljung-Box statistics for the joint hypotheses of no serial correlation among the first four residuals.

[e] Entries in the brackets of this column are the t-statistics for the null hypothesis $\rho_1 = 0$.

[f] Entries in the brackets of this column are the t-statistics for the null hypothesis $\rho_2 = 0$.

CHAPTER 5. ESTIMATION RESULTS

This chapter reports the estimation results of the three threshold cointegration models introduced in Chapter 3.

5.1. Results from the Lo-Zivot Three-Regime Model

5.1.1. Introduction

The models of threshold cointegration were estimated and tested using the monthly (1960:1-1977:12) interest rate series described previously in this research. The bivariate systems of Aaa or Baa corporate bond interest rate indices relative to a benchmark rate were considered. Either the 10-year Treasury note index or the Ibboston corporate bond index was selected as the benchmark. Let $x_{1,t}$ be the Moody's (Aaa or Baa) corporate bond index and let $x_{2,t}$ be the benchmark rate. With two corporate bond indices and two benchmark rates, there are four bivariate systems. The results reported in the data description section suggested that $x_{1,t}$ and $x_{2,t}$ are cointegrated with a cointegratng vector [1, -1]'. Accordingly, for each bivariate system one may construct the rate differential (i.e., cointegrating residual), defined as

$$w_{t-1} = \beta' X_{t-1} = x_{1,t-1} - x_{2,t-1}, \qquad (5.1.1)$$

where $X_t' = (x_{1,t}, x_{2,t})$.

5.1.2. Empirical results: Model selection

Since the goal was to determine whether a three-regime model could describe credit dynamics, the focus was on the alternative hypothesis of a TVECM(3) model. Tables 5.1.1

and 5.1.2 summarize the results from estimating the TVECM(3) model for four bivariate systems with lag specifications equal to one and two, respectively, and test results of this model against the null of a simple VECM. Since the threshold effect has content only if each regime has a minimum number of observations, the restriction was imposed that the threshold values of w_{t-1} fall between the 15-th and 85-th percentiles of w_{t-1}. Then, the grid was searched as defined by all points implied by the data. Details of the estimation strategy are presented in Appendix A.

In Tables 5.1.1 and 5.1.2, "1st regime obs" (2nd regime obs, and 3rd regime obs) report the number of observations that are estimated to fall into the first regime (the second regime, and the third regime, respectively). "C1 hat" and "c2 hat" are the estimated threshold values (i.e., $\hat{c}_1$ and $\hat{c}_2$).

Hansen's (1997) log-likelihood test was applied to test the null hypothesis of a linear VECM against the alternative of a TVECM(3). $LR_{1,3}$ is the sup-LR test statistic. The "p-value" represents the p-value that is calculated by the fixed regressor bootstrap method described by Hansen and Seo (2002). All p-values were computed with 1000 simulated replications[6].

Tables 5.1.1 and 5.1.2 indicate that the estimated thresholds for Ibb vs. Aaa are nearly the same for both lag lengths. The estimated thresholds are lower for the Tsy vs. Aaa than for the Tsy vs. Baa and they are lower for the Ibb vs. Aaa than for the Ibb vs. Baa. In general, they are higher for the Tsy vs. Aaa than for the Ibb vs. Aaa, and they are higher for the Tsy vs. Baa than for the Ibb vs. Baa. The null of no threshold effects cannot be rejected (at conventional significance levels) for the Ibb vs. Aaa or the Ibb vs. Baa pairs for either lag

[6] The results are also reported with 5000 simulation replications for sensitivity testing.

length. The no threshold null cannot be rejected for the Tsy vs. Aaa or Tsy vs. Baa pairs for lag length one, but the null of no threshold effects can be rejected for the Tsy vs. Aaa or Tsy vs. Baa pairs, when the lag length is two. The large increases in the LR statistic that occur in each case, when the lag length increases from one to two, suggest that there are important adjustment dynamics that are not captured by the first-order model. Focusing on the second-order model, the evidence for the TVECM(3) is mixed: it is favorable for the Tsy vs. Aaa and Tsy vs. Baa pairs, but is unfavorable for the Ibb vs. Aaa and Ibb vs. Baa pairs.

5.1.3. Empirical results: Estimated equations of the TVECM(3)

This section reports and discusses the estimates of the four TVECM(3)s, estimated with lag lengths one and two.

Part A. Lag length set to one (in levels):

(i) For Tsy vs. Aaa, the estimated TVECM(3) is given in equations (5.1.2) and (5.1.3):

$$\Delta r_{Aaa,t} = \begin{cases} \underset{(0.16)}{-0.39} + \underset{(0.21)}{0.47} w_{t-1} + u_{1t}, & w_{t-1} \le 0.54, \\ \underset{(0.04)}{0.11} - \underset{(0.12)}{0.16} w_{t-1} + u_{2t}, & 0.54 < w_{t-1} \le 0.95, \\ \underset{(0.17)}{-0.04} - \underset{(0.14)}{0.00} w_{t-1} + u_{3t}, & w_{t-1} > 0.95, \end{cases} \tag{5.1.2}$$

and

$$\Delta r_{Tsy,t} = \begin{cases} \underset{(0.05)}{0.02} + \underset{(0.15)}{0.08} w_{t-1} + u_{1t}, & w_{t-1} \le 0.54, \\ \\ \underset{(0.21)}{-0.22} + \underset{(0.18)}{0.16} w_{t-1} + u_{2t}, & 0.54 < w_{t-1} \le 0.95, \\ \\ \underset{(0.13)}{-0.24} + \underset{(0.16)}{0.28} w_{t-1} + u_{3t}, & w_{t-1} > 0.95. \end{cases} \tag{5.1.3}$$

(ii) For Tsy vs. Baa, the estimated TVECM(3) is given in equations (5.1.4) and (5.1.5):

$$\Delta r_{Baa,t} = \begin{cases} \underset{(0.13)}{0.19} - \underset{(0.08)}{0.12} w_{t-1} + u_{1t}, & w_{t-1} \le 1.21, \\ \\ \underset{(0.08)}{-0.06} + \underset{(0.09)}{0.14} w_{t-1} + u_{2t}, & 1.21 < w_{t-1} \le 2.13, \\ \\ \underset{(0.14)}{0.46} - \underset{(0.05)}{0.20} w_{t-1} + u_{3t}, & w_{t-1} > 2.13, \end{cases} \tag{5.1.4}$$

and

$$\Delta r_{Tsy,t} = \begin{cases} \underset{(0.12)}{-0.07} + \underset{(0.12)}{0.12} w_{t-1} + u_{1t}, & w_{t-1} \le 1.21, \\ \\ \underset{(0.21)}{0.32} - \underset{(0.08)}{0.13} w_{t-1} + u_{2t}, & 1.21 < w_{t-1} \le 2.13, \\ \\ \underset{(0.09)}{0.14} - \underset{(0.05)}{0.08} w_{t-1} + u_{3t}, & w_{t-1} > 2.13. \end{cases} \tag{5.1.5}$$

(iii) For Ibb vs. Aaa, the estimated TVECM(3) is given in equations (5.1.6) and (5.1.7):

$$\Delta r_{Aaa,t} = \begin{cases} \underset{(0.15)}{0.09} - \underset{(0.28)}{0.24} w_{t-1} + u_{1t}, & w_{t-1} \le 0.40, \\ \underset{(0.04)}{0.21} - \underset{(0.13)}{0.48} w_{t-1} + u_{2t}, & 0.40 < w_{t-1} \le 0.70, \\ \underset{(0.08)}{0.24} - \underset{(0.07)}{0.35} w_{t-1} + u_{3t}, & w_{t-1} > 0.70, \end{cases} \quad (5.1.6)$$

and

$$\Delta r_{Ibb,t} = \begin{cases} \underset{(0.05)}{0.01} + \underset{(0.19)}{0.07} w_{t-1} + u_{1t}, & w_{t-1} \le 0.40, \\ \underset{(0.12)}{0.10} - \underset{(0.11)}{0.07} w_{t-1} + u_{2t}, & 0.40 < w_{t-1} \le 0.70, \\ \underset{(0.10)}{0.23} - \underset{(0.19)}{0.43} w_{t-1} + u_{3t}, & w_{t-1} > 0.70. \end{cases} \quad (5.1.7)$$

(iv) For Ibb vs. Baa, the estimated TVECM(3) is given in equations (5.1.8) and (5.1.9):

$$\Delta r_{Baa,t} = \begin{cases} \underset{(0.18)}{-0.16} + \underset{(0.11)}{0.11} w_{t-1} + u_{1t}, & w_{t-1} \le 1.28, \\ \underset{(0.07)}{0.01} + \underset{(0.07)}{0.06} w_{t-1} + u_{2t}, & 1.28 < w_{t-1} \le 2.02, \\ \underset{(0.12)}{0.36} - \underset{(0.05)}{0.18} w_{t-1} + u_{3t}, & w_{t-1} > 2.02, \end{cases} \quad (5.1.8)$$

and

$$\Delta r_{Ibb,t} = \begin{cases} -0.03 + 0.05w_{t-1} + u_{1t}, & w_{t-1} \le 1.28, \\ (0.11)\ (0.11) & \\ -0.26 + 0.09w_{t-1} + u_{2t}, & 1.28 < w_{t-1} \le 2.02, \\ (0.19)\ (0.07) & \\ -0.02 + 0.01w_{t-1} + u_{3t}, & w_{t-1} > 2.02. \\ (0.11)\ (0.07) & \end{cases} \tag{5.1.9}$$

Part B. Lag length set to two (in levels):

(i) For Tsy vs. Aaa, the estimated TVECM(3) is given in equations (5.1.10) and (5.1.11):

$$\Delta r_{Aaa,t} = \begin{cases} 0.21 - 0.43w_{t-1} - 0.19\Delta r_{Aaa,t-1} + 0.59\Delta r_{Tsy,t-1} + u_{1t}, & w_{t-1} \le 0.39, \\ (0.14)\ (0.24)\quad (0.21)\qquad (0.17) & \\ 0.06 - 0.07w_{t-1} - 0.93\Delta r_{Aaa,t-1} + 1.03\Delta r_{Tsy,t-1} + u_{2t}, & 0.39 < w_{t-1} \le 0.74, \\ (0.05)\ (0.0)\quad (0.21)\qquad (0.18) & \\ 0.02 - 0.02w_{t-1} - 0.02\Delta r_{Aaa,t-1} + 0.31\Delta r_{Tsy,t-1} + u_{3t}, & w_{t-1} > 0.74, \\ (0.07)\ (0.07)\quad (0.15)\qquad (0.11) & \end{cases} \tag{5.1.10}$$

and

$$\Delta r_{Tsy,t} = \begin{cases} -0.04 + 0.30w_{t-1} - 1.08\Delta r_{Aaa,t-1} + 1.09\Delta r_{Tsy,t-1} + u_{1t}, & w_{t-1} \le 0.39, \\ (0.07)\ (0.27)\quad (0.28)\qquad (0.24) & \\ -0.04 + 0.06w_{t-1} - 0.07\Delta r_{Aaa,t-1} + 0.38\Delta r_{Tsy,t-1} + u_{2t}, & 0.39 < w_{t-1} \le 0.74, \\ (0.10)\ (0.09)\quad (0.21)\qquad (0.16) & \\ 0.20 - 0.38w_{t-1} - 0.05\Delta r_{Aaa,t-1} + 0.50\Delta r_{Tsy,t-1} + u_{3t}, & w_{t-1} > 0.74. \\ (0.10)\ (0.17)\quad (0.15)\qquad (0.13) & \end{cases} \tag{5.1.11}$$

(ii) For Tsy vs. Baa, the estimated TVECM(3) is given in equations (5.1.12) and (5.1.13):

$$\Delta r_{Baa,t} = \begin{cases} \underset{(0.38)}{0.21} - \underset{(0.20)}{0.15} w_{t-1} - \underset{(0.21)}{1.03} \Delta r_{Baa,t-1} + \underset{(0.15)}{0.99} \Delta r_{Tsy,t-1} + u_{1t}, & w_{t-1} \le 1.68, \\ \underset{(0.04)}{0.01} + \underset{(0.04)}{0.00} w_{t-1} + \underset{(0.12)}{0.14} \Delta r_{Baa,t-1} + \underset{(0.09)}{0.42} \Delta r_{Tsy,t-1} + u_{2t}, & 1.68 < w_{t-1} \le 2.13, \\ \underset{(0.13)}{0.21} - \underset{(0.05)}{0.09} w_{t-1} + \underset{(0.10)}{0.20} \Delta r_{Baa,t-1} + \underset{(0.07)}{0.16} \Delta r_{Tsy,t-1} + u_{3t}, & w_{t-1} > 2.13, \end{cases} \tag{5.1.12}$$

and

$$\Delta r_{Tsy,t} = \begin{cases} -\underset{(0.07)}{0.03} + \underset{(0.06)}{0.04} w_{t-1} - \underset{(0.19)}{0.12} \Delta r_{Baa,t-1} + \underset{(0.14)}{0.43} \Delta r_{Tsy,t-1} + u_{1t}, & w_{t-1} \le 1.68, \\ \underset{(0.20)}{0.09} - \underset{(0.08)}{0.02} w_{t-1} - \underset{(0.16)}{0.10} \Delta r_{Baa,t-1} + \underset{(0.11)}{0.36} \Delta r_{Tsy,t-1} + u_{2t}, & 1.68 < w_{t-1} \le 2.13, \\ \underset{(0.24)}{0.29} - \underset{(0.13)}{0.17} w_{t-1} - \underset{(0.13)}{0.18} \Delta r_{Baa,t-1} + \underset{(0.10)}{0.50} \Delta r_{Tsy,t-1} + u_{3t}, & w_{t-1} > 2.13. \end{cases} \tag{5.1.13}$$

(iii) For Ibb vs. Aaa, the estimated TVECM(3) is given in equations (5.1.14) and (5.1.15):

$$\Delta r_{Aaa,t} = \begin{cases} \underset{(0.16)}{0.10} - \underset{(0.28)}{0.25} w_{t-1} + \underset{(0.12)}{0.01} \Delta r_{Aaa,t-1} - \underset{(0.12)}{0.02} \Delta r_{Ibb,t-1} + u_{1t}, & w_{t-1} \le 0.40, \\ \underset{(0.03)}{0.04} - \underset{(0.11)}{0.06} w_{t-1} + \underset{(0.10)}{0.19} \Delta r_{Aaa,t-1} + \underset{(0.07)}{0.46} \Delta r_{Ibb,t-1} + u_{2t}, & 0.40 < w_{t-1} \le 0.70, \\ \underset{(0.07)}{0.09} - \underset{(0.06)}{0.10} w_{t-1} - \underset{(0.06)}{0.11} \Delta r_{Aaa,t-1} + \underset{(0.05)}{0.54} \Delta r_{Ibb,t-1} + u_{3t}, & w_{t-1} > 0.70, \end{cases} \tag{5.1.14}$$

and

$$
\Delta r_{Ibb,t} = \begin{cases} \underset{(0.06)}{-0.06} + \underset{(0.21)}{0.24} w_{t-1} + \underset{(0.18)}{0.14} \Delta r_{Aaa,t-1} + \underset{(0.13)}{0.14} \Delta r_{Ibb,t-1} + u_{1t}, & w_{t-1} \le 0.40, \\ \underset{(0.12)}{0.03} + \underset{(0.12)}{0.03} w_{t-1} - \underset{(0.12)}{0.13} \Delta r_{Aaa,t-1} + \underset{(0.09)}{0.22} \Delta r_{Ibb,t-1} + u_{2t}, & 0.40 < w_{t-1} \le 0.70, \\ \underset{(0.09)}{0.08} - \underset{(0.15)}{0.16} w_{t-1} + \underset{(0.07)}{0.01} \Delta r_{Aaa,t-1} + \underset{(0.07)}{0.38} \Delta r_{Ibb,t-1} + u_{3t}, & w_{t-1} > 0.70. \end{cases} \quad (5.1.15)
$$

(iv) For Ibb vs. Baa, the estimated TVECM(3) is given in equations (5.1.16) and (5.1.17):

$$
\Delta r_{Baa,t} = \begin{cases} \underset{(0.14)}{-0.04} + \underset{(0.10)}{0.03} w_{t-1} + \underset{(0.11)}{0.03} \Delta r_{Baa,t-1} + \underset{(0.09)}{0.27} \Delta r_{Ibb,t-1} + u_{1t}, & w_{t-1} \le 1.14, \\ \underset{(0.06)}{0.01} + \underset{(0.07)}{0.02} w_{t-1} + \underset{(0.12)}{0.26} \Delta r_{Baa,t-1} + \underset{(0.08)}{0.33} \Delta r_{Ibb,t-1} + u_{2t}, & 1.14 < w_{t-1} \le 1.94, \\ \underset{(0.08)}{0.02} - \underset{(0.03)}{0.03} w_{t-1} + \underset{(0.06)}{0.09} \Delta r_{Baa,t-1} + \underset{(0.04)}{0.34} \Delta r_{Ibb,t-1} + u_{3t}, & w_{t-1} > 1.94, \end{cases} \quad (5.1.16)
$$

and

$$
\Delta r_{Ibb,t} = \begin{cases} \underset{(0.13)}{0.02} + \underset{(0.14)}{0.00} w_{t-1} + \underset{(0.26)}{0.13} \Delta r_{Baa,t-1} - \underset{(0.17)}{0.14} \Delta r_{Ibb,t-1} + u_{1t}, & w_{t-1} \le 1.14, \\ \underset{(0.18)}{-0.07} + \underset{(0.07)}{0.02} w_{t-1} - \underset{(0.12)}{0.34} \Delta r_{Baa,t-1} + \underset{(0.08)}{0.08} \Delta r_{Ibb,t-1} + u_{2t}, & 1.14 < w_{t-1} \le 1.94, \\ \underset{(0.07)}{-0.04} + \underset{(0.05)}{0.03} w_{t-1} + \underset{(0.05)}{0.26} \Delta r_{Baa,t-1} + \underset{(0.04)}{0.46} \Delta r_{Ibb,t-1} + u_{3t}, & w_{t-1} > 1.94. \end{cases} \quad (5.1.17)
$$

5.1.4. Conclusions

In summary, with lagged term set to two in levels, one generally finds evidence for asymmetry in the adjustment of interest rates. For example, Equations (5.1.12) and (5.1.13) summarize the estimated speed of adjustment parameters for the TVECM(3) of the pair Tsy vs. Baa. The estimated coefficient of the error-correction term in the third-regime for benchmark rate (Tsy) is relatively large and negative (-0.17), while the estimate in the third-regime of the error-correction term for corporate bond index (Baa) is also negative (-0.09). Moreover, the estimated coefficient of the error-correction term in the first-regime for Tsy is positive (0.04), but the estimate in the first-regime of the error-correction term for Baa is negative (-0.15). Apparently, rates in the benchmark tend to "catch up" to rates in the corporate bond index in response to deviations, while the rates in corporate bond index are less affected.

For lag specification one in levels, all of the long rate equations (Aaa or Baa) in the third-regime have the expected signs (negative) for their error correction terms coefficients. That is, the long rates (Aaa, Baa) tend to adjust downward to the equilibrium band when the spread (i.e., the cointegrating residual) is greater than the second (and the larger) estimated threshold. This tendency is not as noteworthy for the short rate (Tsy or Ibb) equations (two of the coefficients of error correction terms are negative, while two of them are positive.)

In addition, for lag specification two in levels, all of the long rate equations (Aaa or Baa) in the third-regime have the expected negative signs for their error correction terms coefficients. In contrast, the short rate (Ibb) equation (5.1.17) shows that the error correction term coefficient in the third regime is the only one error correction term coefficient that violates the negative sign expectation.

The aforementioned eight TVECM(3)s are used to conduct forecasting performance evaluations in Chapter 6.

Table 5.1.1. Lo-Zivot 3-regime model with lag = 1 in levels

	Tsy vs. Aaa	Tsy vs. Baa	Ibb vs. Aaa	Ibb vs. Baa
1st regime obs	178	114	169	177
2nd regime obs	171	236	167	178
3rd regime obs	106	105	119	100
C1 hat	0.54	1.21	0.40	1.28
C2 hat	0.95	2.13	0.70	2.02
$LR_{1,3}$	20.67	20.53	12.08	18.84
p-value*	0.197	0.218	0.833	0.345
p-value**	0.188	0.231	0.832	0.349

Note:
* Fixed regressor bootstrap with 1000 simulation replications.
** Fixed regressor bootstrap with 5000 simulation replications.

Table 5.1.2. Lo-Zivot 3-regime model with lag = 2 in levels

	Tsy vs. Aaa	Tsy vs. Baa	Ibb vs. Aaa	Ibb vs. Baa
1st regime obs	121	234	168	131
2nd regime obs	131	115	167	212
3rd regime obs	202	105	119	111
C1 hat	0.39	1.68	0.40	1.14
C2 hat	0.74	2.13	0.70	1.94
$LR_{1,3}$	43.89	53.90	20.02	26.00
p-value*	0.022	0.000	0.869	0.567
p-value**	0.017	0.002	0.861	0.559

Note:
* Fixed regressor bootstrap with 1000 simulation replications.
** Fixed regressor bootstrap with 5000 simulation replications.

5.2. Results from the Hansen-Seo Two-Regime Model

5.2.1. Introduction

Let $r_{a,t}$ be the Treasury rate (or the Ibboston bond index) and $r_{b,t}$ be the corporate Aaa (or Baa) bond index. The results of the data description section suggest that $r_{a,t}$ and $r_{b,t}$ are cointegrated. A linear cointegrating VAR model has the vector error-correction representation:

$$\begin{pmatrix} \Delta r_{b,t} \\ \Delta r_{a,t} \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} \alpha_1 \\ \alpha_2 \end{pmatrix} (r_{b,t-1} - \beta r_{a,t-1}) + \begin{pmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{pmatrix} \begin{pmatrix} \Delta r_{b,t-1} \\ \Delta r_{a,t-1} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix}, \tag{5.2.1}$$

where $\Gamma_{i,j}(L)$ is a polynomial in the lag operator L for i, j = 1, 2. Here, if one sets $\beta = 1$, then the error-correction term becomes the usual interest rate spread.

However, as suggested previously, linearity may be an inappropriate assumption. This section reports the estimates obtained from fitting a two-regime threshold cointegration model to describe the dynamics of the Treasury rate and the corporate bond index.

The models of threshold cointegration were estimated and tested using the monthly interest series of Neil, Ralph and Morris (2000, hereafter NRM). After NRM, the following period was used: 1960:1 to 1997:12, which totaled 456 monthly observations. For the selection of the VAR lag length, it was noticed that the AIC and BIC consistently selected a lag length equal to two for the pair of Tsy and Aaa. This was the lag length selected for both the linear VECM and the threshold VECM, whether the cointegration parameter β was fixed at unity or estimated from the data. However, for the other interest rate pairs, the lag selection process recommends a lag length equal to one. For robustness, the results for both one-lag and two-lag TVECMs are reported.

5.2.2. Estimation results

Table 5.2.1 summarizes the main results from fitting the threshold VECM to the four interest rate spreads for lag lengths one and two months and for β set to one and for β estimated from the data. In addition to reporting the AIC and BIC for these estimated models, Table 5.2.1 includes the following information. The estimated threshold value for each spread is listed on the line labeled "Gamma hat", while the estimated β is listed in Part B of Table 5.2.1, on the line labeled "Beta hat." Note that the cointegrating relationships are normalized with the coefficient on Aaa and Baa bond rates set to unity, so that β-hat is the estimated coefficient on Tsy or Ibb in the cointegrating relationship. The entries on line labeled "1st regime obs" denote the proportion of observations for which $r_{b,t} - \hat{\beta} r_{at} \leq \hat{\gamma}$ and the entries in the line labeled "2nd regime obs" denote the proportion of observations for which $r_{b,t} - \hat{\beta} r_{at} > \hat{\gamma}$. The entries on the lines labeled "p-value" refer to p-values for the test of the null of linearity against the alternative of two-regime threshold nonlinearity. These results will be discussed in more detail as follows. For comparison purposes, the results are presented from fitting the linear VECM model in Table 5.2.2. Notice in Table 5.2.2 that all the Engle-Granger test-statistics are greater (in absolute value) than the critical value 2.87 for 500 observations at the 5% significance level, indicating the presence of cointegration between each pair of interest rates considered.

The SupLM^0 test (with $\beta = 1$) and SupLM test (with β estimated) developed by Hansen and Seo (2002) were used to test the null of linear cointegration against the threshold cointegration alternative, by calculating the p-values by both the fixed regressor bootstrap and residual bootstrap methods described by Hansen and Seo (2002)[7].

The grid search begun with 100 grid points, then re-estimated the model using 300 grid points for the threshold variables. The rationale was that there are 456 total observations; by excluding lags, and difference terms, there are 452 usable observations. The total number of the threshold candidates is approximately 300 after excluding the top and bottom 15% of the ordered threshold variables, and after taking into account the possible same values of the threshold variables. If one selects 100 grid points, then one out of three possible threshold candidates are evaluated. By selecting 300 grid points, approximately every possible threshold candidate is evaluated if the threshold variables are uniformly distributed in the 70% middle range of the selection. The search range of β is defined as the linear consistent estimate of β, estimated from the linear cointegration model pluses and minuses 6 times of the estimated standard error of β. Next selected were 300 evenly spaced data points in this search range as the possible candidates of β. After forming the grid points of threshold variable and the cointegrating variable, a two-dimensional grid search was conducted to find the values that maximize the likelihood function. The estimation procedures are described in Appendix B. Notice that searching too many grid points will increase computational burden and will make the model unrealistic.

Table 5.2.1 also reports the p-values calculated from the fixed regressor bootstrap and the residual bootstrap methods. All p-values were computed with 300 grid points searching over the threshold variable and the cointegrating variable, with 1000 simulated replications.

---

[7] See Appendix B for more details on the Hansen-Seo estimation and test procedures.

5.2.3. Sensitivity testing

Two robustness checks were conducted to examine the sensitivity of the results. First, the model was estimated with 300 grid points and 5000 simulated replications. The calculated p-values are also displayed in Table 5.2.1. Second, Tables 5.2.3 to 5.2.10 display the estimation results of all eight models with grid points equal to 300 and 100, respectively. The results are reasonably stable to the grid points that were selected. One should note that model selection criterion AIC selects the model with grid points equal 300 over the counterpart of grid points equal 100 for all models. Thus, in the following section, the discussion focuses on the results from the estimated model with 300 grid points.

5.2.4. Other results

The results from the threshold cointegration tests differ drastically depending upon whether Tsy or Ibb is used to measure Treasury rates. Linear cointegration is rejected at the 10% level for all cases using Tsy, except for Tsy vs. Baa with two lags and β fixed at unity. Nevertheless, the p-values are relatively small (14% or 21%), depending on the method used to construct the p-value). On the other hand, the p-values for the test using the Ibb-Aaa and Ibb-Baa spreads are all relatively large. Interestingly, when lag = 2 is set, the evidence for threshold cointegration appears to weaken, with only one of the four significant at the 10% level. However, if β is estimated freely, the evidence for threshold cointegration is strengthened; with two of four significant at the 10% level in either lag specification. In general, the p-values estimated from the residual bootstrap method appear to be larger than the p-values estimated from the fixed regressor bootstrap method, with only three exceptions (Ibb vs. Baa with one lag and Ibb vs. Aaa for both lag specifications).

Next to be addressed are the coefficient estimates for the TVECM models for each of the four pairs of interest rates (Tsy-Aaa, Tsy-Baa, Ibb-Aaa, Ibb-Baa). The model is estimated by minimizing the concentrated likelihood equation (5.2.7), as given by Hansen-Seo (2002) (see Appendix B for details of the estimation procedure).

First, a detailed discussion is provided regarding the estimated TVECMs for the relationship between the Treasury rate and the Baa bond index, one in which the cointegrating parameter β is estimated and the other in which β is fixed at unity. First, consider the model with β estimated.

The estimated cointegration relationship is $w_t = r_{Baa,t} - 1.108 r_{Tsy,t}$. The β estimate is approximately 10.8% greater than a unit coefficient. The estimated threshold is $\hat{\gamma} = 0.351$, which is expressed in terms of percent per year. Hence, the first regime occurs when $r_{Baa,t} \leq 1.108 r_{Tsy,t} + 0.351$, i.e., when the Baa bond index is less than 0.351 percentage points above the Treasury rate (after appropriate adjustment through cointegrating relationship). Seventeen percent of the observations fall into this regime, which Hansen and Seo call the "extreme" regime, since less than half the observations fall into this regime. The second regime is when $r_{Baa,t} > 1.108 r_{Tsy,t} + 0.351$, where the Baa bond index is more than 0.351 percentage points above the Treasury rate. Approximately 83% of the observations fall into this regime, which Hansen and Seo would call the "typical" regime.

The estimated threshold VECM is given in equations (5.2.2) and (5.2.3):

$$\Delta r_{Baa,t} = \begin{cases} \underset{(0.02)}{0.04} - \underset{(0.14)}{0.09} w_{t-1} - \underset{(0.16)}{0.54} \Delta r_{Baa,t-1} + \underset{(0.16)}{0.92} \Delta r_{Tsy,t-1} + u_{1t}, & w_{t-1} \leq 0.351, \\ \\ \underset{(0.02)}{0.03} - \underset{(0.02)}{0.04} w_{t-1} + \underset{(0.11)}{0.22} \Delta r_{Baa,t-1} + \underset{(0.09)}{0.22} \Delta r_{Tsy,t-1} + u_{1t}, & w_{t-1} > 0.351, \end{cases} \tag{5.2.2}$$

and

$$\Delta r_{Tsy,t} = \begin{cases} \underset{(0.05)}{0.02} - \underset{(0.26)}{0.03} w_{t-1} - \underset{(0.40)}{1.53} \Delta r_{Baa,t-1} + \underset{(0.35)}{1.32} \Delta r_{Tsy,t-1} + u_{2t}, & w_{t-1} \leq 0.351, \\ \\ -\underset{(0.03)}{0.01} + \underset{(0.03)}{0.02} w_{t-1} - \underset{(0.16)}{0.04} \Delta r_{Baa,t-1} + \underset{(0.13)}{0.39} \Delta r_{Tsy,t-1} + u_{2t}, & w_{t-1} > 0.351. \end{cases} \tag{5.2.3}$$

Notice that all the point estimates in the extreme regime (when $w_{t-1} \leq 0.351$) are larger (in absolute value) than the corresponding parameters in the typical regime. In fact, the coefficients in the equations for the typical regime are small enough to suggest that in this regime $\Delta r_{Baa,t}$ and $\Delta r_{Tsy,t}$ are close to driftless random walks, or simple AR(1) models. Eicker-White standard errors are presented in parenthesis; however, these should be interpreted with caution, since there is no formal distribution theory for the parameter estimates and standard errors[8].

[8] There are occasions in econometric modeling when the assumption of constant error variance, or *homoscedasticity*, is unreasonable. For example, one might assume that error terms associated with one variable (Aaa or Baa) might have larger variances than those error terms associated with the other variable (Tsy or Ibb). With heteroscedasticity, biased and inconsistent estimation of the variances of the ordinary least-squares parameter estimates causes statistical inference to be invalid. Halbert White has suggested a method for obtaining consistent estimates of variances and covariances of OLS estimates, which provides valid statistical tests for large sample. White's heteroscedasticity-consistent estimator (HCE) is based on the principle of maximum likelihood. With HCE estimation, the R^2 for the regression will be the same, but all estimates of standard errors and related statistics will change because they are now consistent estimates. In particular, the variances will be larger than the variances associated with OLS estimators. However, HCE does not provide the most efficient parameter estimates. (For efficient estimation, one of the weighted least-squares estimation procedures must be used.)

Next reported are the parameter estimates of the model, when the cointegration parameter β is fixed at unity. The threshold estimate is 1.410 (percent per year). Thirty-six percent of the observations fall into the extreme regime and 64% fall into the typical regime. Here, the "extreme" regime occurs when $r_{Baa,t} \le r_{Tsy,t} + 1.410$, while the "typical" regime occurs when $r_{Baa,t} > r_{Tsy,t} + 1.410$. The estimated threshold VECM is given in equations (5.2.4) and (5.2.5):

$$\Delta r_{Baa,t} = \begin{cases} \underset{(0.02)}{0.01} + \underset{(0.02)}{0.01} w_{t-1} - \underset{(0.09)}{0.02} \Delta r_{Baa,t-1} + \underset{(0.07)}{0.54} \Delta r_{Tsy,t-1} + u_{1t}, & w_{t-1} \le 1.410, \\ \\ \underset{(0.05)}{0.05} - \underset{(0.03)}{0.03} w_{t-1} + \underset{(0.12)}{0.13} \Delta r_{Baa,t-1} + \underset{(0.10)}{0.26} \Delta r_{Tsy,t-1} + u_{1t}, & w_{t-1} > 1.410, \end{cases} \tag{5.2.4}$$

and

$$\Delta r_{Tsy,t} = \begin{cases} -\underset{(0.03)}{0.03} + \underset{(0.04)}{0.06} w_{t-1} - \underset{(0.17)}{0.30} \Delta r_{Baa,t-1} + \underset{(0.12)}{0.42} \Delta r_{Tsy,t-1} + u_{2t}, & w_{t-1} \le 1.410, \\ \\ -\underset{(0.08)}{0.06} + \underset{(0.04)}{0.03} w_{t-1} - \underset{(0.22)}{0.32} \Delta r_{Baa,t-1} + \underset{(0.15)}{0.54} \Delta r_{Tsy,t-1} + u_{2t}, & w_{t-1} > 1.410. \end{cases} \tag{5.2.5}$$

The finding of previous two specifications is of great interest. Comparing equations (5.2.2) and (5.2.3) to the pair of (5.2.4) and (5.2.5), in the extreme regime equations, the signs of the error-correction coefficients switch from negative to positive: (-0.09, -0.03) to (0.01, 0.06). As articulated by Campbell and Shiller (1991), and Campbell (1995), as well as the work by Hansen and Seo (2002), the coefficient of the error-correction term w_{t-1}, in the threshold VECM, should be positive. The rationale behind this argument stems from the term structure of interest rates. Since a large positive spread $(r_{Baa,t} - r_{Tsy,t})$ implies the long bond ($r_{Baa,t}$) earns a higher rate than the expected interest rate, the long bond must be expected to depreciate in value. Hence, the long interest rate is expected to rise; at the same time, however, the short interest rate ($r_{Tsy,t}$) is also expected to rise, since the long rate is a smoothed forecast of future short rates.

Campbell and Shiller (1991) and Campbell (1995) found evidence that changes in the short rate are positively correlated with the spread, whereas changes in the long rate are negatively correlated with the spread. These authors called this finding a "puzzle", since it is against the prediction of the term structure theory. However, Hansen and Seo (2002) found support for the prediction of the term structure theory. In their estimation, the coefficients of the error-correction terms are either positive or not significantly different from zero, if negative. So, there appears to be no puzzle to Hansen and Seo. The current threshold cointegration approach, under the β-equals-unity case, supports the argument of Hansen and Seo (2002), since all of the coefficients of error-correction terms in the extreme regime are positive. However, in the case of estimated-β, both long and short rate error-correction coefficients are negative (although not too significant for the short rate equation), which is similar to the finding of Campbell and Shiller (1991) and Campbell (1995).

Last reported is the conventional linear VECM as follows:

(1) With the cointegrating vector estimated from the model, we obtain $\hat{\beta}$ = 1.185, so that $w_t = r_{Baa,t} - 1.185 r_{Tsy,t}$. The error-correction representation is given in equations (5.2.6) and (5.2.7):

$$\Delta r_{Baa,t} = \begin{cases} 0.01 - 0.03 w_{t-1} + 0.11 \Delta r_{Baa,t-1} + 0.30 \Delta r_{Tsy,t-1} + u_{1t}, \\ (0.01)\ (0.02) \qquad (0.10) \qquad\qquad (0.09) \end{cases} \tag{5.2.6}$$

$$\Delta r_{Tsy,t} = \begin{cases} -0.01 + 0.04 w_{t-1} - 0.29 \Delta r_{Baa,t-1} + 0.52 \Delta r_{Tsy,t-1} + u_{2t}. \\ (0.02)\ (0.03) \quad\ (0.18) \qquad\qquad (0.13) \end{cases} \tag{5.2.7}$$

(2) With the cointegration vector fixed at [1, -1]' the error correction representation is given in equations (5.2.8) and (5.2.9):

$$\Delta r_{Baa,t} = \begin{cases} 0.06 - 0.03 w_{t-1} + 0.12 \Delta r_{Baa,t-1} + 0.30 \Delta r_{Tsy,t-1} + u_{1t}, \\ (0.02)\ (0.01) \quad\ (0.10) \qquad\qquad (0.09) \end{cases} \tag{5.2.8}$$

$$\Delta r_{Tsy,t} = \begin{cases} 0.01 - 0.00 w_{t-1} - 0.32 \Delta r_{Baa,t-1} + 0.51 \Delta r_{Tsy,t-1} + u_{2t}. \\ (0.03)(0.02) \quad\ (0.19) \qquad\qquad (0.13) \end{cases} \tag{5.2.9}$$

Notice that the estimated models (5.2.6)-(5.2.7) and (5.2.8)-(5.2.9) are nearly the same. This appears to be the case because the error-correction term has little effect in any of the four equations. Consequently, the value specified for β does not affect the model's dynamics.

One may compare the dynamics shown in equations (5.2.2)-(5.2.3) (threshold cointegration model) versus those in equations (5.2.6)-(5.2.7) (linear cointegration model) for the case of β is estimated from the data and equations (5.2.4)-(5.2.5) (threshold cointegration model) versus equations (5.2.8)-(5.2.9) (linear cointegration model) for β is fixed at unity.

For the $\Delta r_{Baa,t}$ equations (5.2.2) and (5.2.6), the signs of the coefficient estimates are in agreement for the constant, w_{t-1}, and $\Delta r_{Tsy,t-1}$ terms. The coefficient estimate of $\Delta r_{Baa,t-1}$ in equation (5.2.2) is negative (-0.54) (with standard error 0.16) under the extreme regime, while it is positive (0.22) (with a standard error 0.11) under the typical regime. In equation (5.2.6), the coefficient estimate of $\Delta r_{Baa,t-1}$ is positive (0.11) (with a standard error 0.10). Hence, there exists a negative impact (with a coefficient estimate of -0.54) in the change of previous period's Baa rates on the change of current Baa rates—under the extreme regime of the threshold cointegration model. Furthermore, given that the standard errors are 0.16 and 0.11 in equation (5.2.2), the t-statistics are -3.375 and 2.000 for the $\Delta r_{Baa,t-1}$ coefficients, respectively; while in equation (5.2.6), the t-statistic is 1.10 for the $\Delta r_{Baa,t-1}$ coefficient.

For the $\Delta r_{Tsy,t}$ equations (5.2.3) and (5.2.7), the signs of the coefficient estimates are in agreement with respect to the $\Delta r_{Baa,t-1}$, and $\Delta r_{Tsy,t-1}$ terms. The coefficient estimate of the constant term in equation (5.2.3) is positive (0.02) under the extreme regime, while it is negative (-0.01) under the typical regime (the t-statistics are 0.40 and -0.33, respectively.) In equation (5.2.7), the coefficient estimate of constant is negative (-0.01) (the t-statistic is – 0.50.) Next, the coefficient estimate of w_{t-1} in equation (5.2.3) is negative (-0.03) under the extreme regime, while it is positive (0.02) under the typical regime (the t-statistics are -0.12 and 0.67, respectively). In equation (5.2.7), the coefficient estimate of w_{t-1} is positive (0.04) (the t-statistic is 1.33).

For the $\Delta r_{Baa,t}$ equations (5.2.4) and (5.2.8), the signs of coefficient estimates are in agreement with constant and $\Delta r_{Tsy,t-1}$ terms. The coefficient estimate of w_{t-1} in equation (5.2.4) is positive (0.01) (with a t-statistic of 0.50) under the extreme regime, while it is negative (-0.03) (with t-statistic of 1.00) under the typical regime. In equation (5.2.8), the coefficient estimate of w_{t-1} is negative (-0.03) (with a t-statistic of -3.00). Next, the coefficient estimate of $\Delta r_{Baa,t-1}$ in equation (5.2.4) is negative (-0.02) (with a t-statistic of - 0.22) under the extreme regime, while it is positive (0.13) (with t-statistic of 1.08) under the typical regime. In equation (5.2.8), the coefficient estimate of $\Delta r_{Baa,t-1}$ is positive (0.12) (with a t-statistic of 1.20).

For the $\Delta r_{Tsy,t}$ equations (5.2.5) and (5.2.9), the signs of coefficient estimates are in agreement with respect to the $\Delta r_{Baa,t-1}$, and $\Delta r_{Tsy,t-1}$ terms, which have the same pattern as the estimates of equations (5.2.3) and (5.2.7) (threshold cointegration model). The coefficient estimate of the constant term in equation (5.2.5) is negative under both regimes (with t-statistics of -1.00 and -0.75, respectively.) In equation (5.2.9), the coefficient estimate of constant term is positive (0.01) (with t-statistic of 0.33.) Next, the coefficient estimate of w_{t-1} in equation (5.2.5) is positive under both regimes (with t-statistics of 1.50 and 0.75, respectively). In equation (5.2.9), the coefficient estimate of w_{t-1} is negative (-0.00) (with a statistic of -0.00).

One may also examine the dynamic adjustments to unit shocks to equations (5.2.2) and (5.2.3) versus equations (5.2.6) and (5.2.7) when the cointegrating parameter is estimated from the model for TVECM and linear VECM of Baa and Tsy pair. The above estimates for equations (5.2.2) and (5.2.3) show that both Baa and Tsy adjust moderately to a unit shock when both rates fall into the second regime (the typical regime, where $w_{t-1} > 0.351$). Within any month, the Tsy rate adjusts roughly 2% and the Baa rate adjusts -4% in response to a positive 1-unit deviation from the long-run equilibrium. When both rates fall into the first regime (the extreme regime, where $w_{t-1} \leq 0.351$), for one unit deviation from the long-run equilibrium, the corporate rates Baa adjust more strongly (-9%) than the Tsy rates (about -3%). Thus, adjustment to the long-run equilibrium shows a strong tendency, when the Baa rates wander away from the cointegration relationship with one unit change in the regime where $w_{t-1} \leq 0.351$.

Within the regime of symmetric adjustment, equations (5.2.6) and (5.2.7) show that for both rates the adjustments to the long-run equilibrium are moderately slow. Overall, for a

1-unit gap away from long-run equilibrium, Tsy and Aaa adjust roughly 4% and -3%, respectively, regardless of the sign of the deviation from long-run equilibrium.

When the cointegration parameter β is fixed at unity, equations (5.2.4) and (5.2.5) show that both Baa and Tsy adjust moderately to a unit change when they fall into the typical regime ($w_{t-1} > 1.410$). Within any month, the Tsy rate adjusts roughly 3% and the Baa rate adjusts -3% in response to a positive 1-unit deviation from the long-run equilibrium. Under the extreme regime ($w_{t-1} \leq 1.410$), the Tsy rates adjust more strongly (about 6%) than the corporate rates Baa (1%). Thus, adjustment to the long-run equilibrium shows a strong tendency when the Tsy rates wander away from the cointegration relationship with one unit change in extreme regime.

Within the regime of symmetric adjustment, these estimates (equations (5.2.8) and (5.2.9)) show that for both rates the adjustments to the long-run equilibrium are relatively slow. Overall, for a 1-unit gap away from long-run equilibrium, Tsy and Aaa adjust roughly -0% and -3%, respectively, regardless of the sign of the deviation from long-run equilibrium.

Two sets of threshold cointegration estimations are reported for each of the other three pairs of interest rates, one with the cointegration parameter β estimated and the other fixing β at unity. The lag length (2 vs. 1), for each case, was selected based on the AIC and BIC presented in Table 5.2.1. A couple of observations from Table 5.2.1 are worth noting at this point. First, whether the first regime or the second regime is the extreme regime depends heavily upon whether the Ibbotson index or Treasury rate is used. Second, in the IBB vs. Aaa case (β estimated from the model), with different lag specification, the number of data points in the first regime flips from one end to the other (15% vs. 79%).

For the pair of Tsy vs. Aaa, with the lag length equals to two and β is estimated from the model, the estimated TVECM is given in equations (5.2.10) and (5.2.11):

$$\Delta r_{Aaa,t} = \begin{cases} \underset{(0.06)}{0.06} - \underset{(0.31)}{0.17} w_{t-1} - \underset{(0.40)}{1.20} \Delta r_{Aaa,t-1} + \underset{(0.27)}{1.20} \Delta r_{Tsy,t-1} - \underset{(0.49)}{0.63} \Delta r_{Aaa,t-2} \\ \qquad + \underset{(0.40)}{0.47} \Delta r_{Tsy,t-2} + u_{1t}, & w_{t-1} \leq 0.203, \\ \\ \underset{(0.03)}{0.04} - \underset{(0.03)}{0.06} w_{t-1} + \underset{(0.12)}{0.21} \Delta r_{Aaa,t-1} + \underset{(0.08)}{0.29} \Delta r_{Tsy,t-1} + \underset{(0.12)}{0.01} \Delta r_{Aaa,t-2} \\ \qquad - \underset{(0.10)}{0.24} \Delta r_{Tsy,t-2} + u_{1t}, & w_{t-1} > 0.203, \end{cases} \tag{5.2.10}$$

and

$$\Delta r_{Tsy,t} = \begin{cases} -\underset{(0.08)}{0.00} + \underset{(0.40)}{0.17} w_{t-1} - \underset{(0.55)}{1.10} \Delta r_{Aaa,t-1} + \underset{(0.41)}{1.09} \Delta r_{Tsy,t-1} - \underset{(0.75)}{0.57} \Delta r_{Aaa,t-2} \\ \qquad + \underset{(0.60)}{0.29} \Delta r_{Tsy,t-2} + u_{2t}, & w_{t-1} \leq 0.203, \\ \\ \underset{(0.03)}{0.01} - \underset{(0.04)}{0.01} w_{t-1} + \underset{(0.17)}{0.12} \Delta r_{Aaa,t-1} + \underset{(0.11)}{0.38} \Delta r_{Tsy,t-1} + \underset{(0.17)}{0.26} \Delta r_{Aaa,t-2} \\ \qquad - \underset{(0.13)}{0.42} \Delta r_{Tsy,t-2} + u_{2t}, & w_{t-1} > 0.203. \end{cases} \tag{5.2.11}$$

For the pair of Tsy vs. Aaa, with the lag length equals to two, the estimated TVECM with fixed cointegrating vector is given in equations (5.2.12) and (5.2.13):

$$\Delta r_{Aaa,t} = \begin{cases} \underset{(0.07)}{0.08} - \underset{(0.34)}{0.19} w_{t-1} - \underset{(0.37)}{1.08} \Delta r_{Aaa,t-1} + \underset{(0.26)}{1.10} \Delta r_{Tsy,t-1} - \underset{(0.49)}{0.54} \Delta r_{Aaa,t-2} \\ \qquad + \underset{(0.36)}{0.38} \Delta r_{Tsy,t-2} + u_{1t}, \qquad w_{t-1} \le 0.280, \\ \\ \underset{(0.03)}{0.04} - \underset{(0.03)}{0.06} w_{t-1} + \underset{(0.12)}{0.22} \Delta r_{Aaa,t-1} + \underset{(0.08)}{0.28} \Delta r_{Tsy,t-1} + \underset{(0.12)}{0.01} \Delta r_{Aaa,t-2} \\ \qquad - \underset{(0.10)}{0.24} \Delta r_{Tsy,t-2} + u_{1t}, \qquad w_{t-1} > 0.280, \end{cases} \tag{5.2.12}$$

and

$$\Delta r_{Tsy,t} = \begin{cases} -\underset{(0.10)}{0.01} + \underset{(0.46)}{0.21} w_{t-1} - \underset{(0.50)}{1.04} \Delta r_{Aaa,t-1} + \underset{(0.39)}{1.04} \Delta r_{Tsy,t-1} - \underset{(0.75)}{0.47} \Delta r_{Aaa,t-2} \\ \qquad + \underset{(0.55)}{0.19} \Delta r_{Tsy,t-2} + u_{2t}, \qquad w_{t-1} \le 0.280, \\ \\ \underset{(0.04)}{0.00} - \underset{(0.05)}{0.00} w_{t-1} + \underset{(0.17)}{0.13} \Delta r_{Aaa,t-1} + \underset{(0.11)}{0.37} \Delta r_{Tsy,t-1} + \underset{(0.17)}{0.24} \Delta r_{Aaa,t-2} \\ \qquad - \underset{(0.13)}{0.41} \Delta r_{Tsy,t-2} + u_{2t}, \qquad w_{t-1} > 0.280. \end{cases} \tag{5.2.13}$$

For the pair of Ibb vs. Aaa, with lag length equals to one and β estimated from the model, the estimated TVECM is given in equations (5.2.14) and (5.2.15):

$$\Delta r_{Aaa,t} = \begin{cases} -\underset{(0.09)}{0.20} + \underset{(0.31)}{0.72} w_{t-1} + \underset{(0.24)}{0.13} \Delta r_{Aaa,t-1} + \underset{(0.19)}{0.84} \Delta r_{Ibb,t-1} + u_{1t}, w_{t-1} \le 0.350, \\ \\ \underset{(0.03)}{0.05} - \underset{(0.04)}{0.07} w_{t-1} - \underset{(0.06)}{0.02} \Delta r_{Aaa,t-1} + \underset{(0.05)}{0.46} \Delta r_{Ibb,t-1} + u_{1t}, \quad w_{t-1} > 0.350, \end{cases} \tag{5.2.14}$$

and

$$\Delta r_{Ibb,t} = \begin{cases} -0.37 + 1.35 w_{t-1} + 0.42 \Delta r_{Aaa,t-1} + 0.27 \Delta r_{Ibb,t-1} + u_{2t},\ w_{t-1} \le 0.350, \\ (0.16)\ (0.53) \qquad (0.51) \qquad\qquad (0.34) \\ \\ -0.04 + 0.05 w_{t-1} - 0.07 \Delta r_{Aaa,t-1} + 0.13 \Delta r_{Ibb,t-1} + u_{2t},\ w_{t-1} > 0.350. \\ (0.05)\ (0.07) \qquad (0.11) \qquad\qquad (0.09) \end{cases} \tag{5.2.15}$$

For the pair of Ibb vs. Aaa, with lag length equals to one, the estimated TVECM with fixed cointegrating vector is given in equations (5.2.16) and (5.2.17):

$$\Delta r_{Aaa,t} = \begin{cases} 0.09 - 0.23 w_{t-1} + 0.05 \Delta r_{Aaa,t-1} + 0.43 \Delta r_{Ibb,t-1} + u_{1t},\quad w_{t-1} \le 0.620, \\ (0.03)(0.08) \qquad (0.08) \qquad\qquad (0.06) \\ \\ 0.11 - 0.11 w_{t-1} - 0.09 \Delta r_{Aaa,t-1} + 0.53 \Delta r_{Ibb,t-1} + u_{1t},\quad w_{t-1} > 0.620, \\ (0.07)\ (0.08) \qquad (0.09) \qquad\qquad (0.07) \end{cases} \tag{5.2.16}$$

and

$$\Delta r_{Ibb,t} = \begin{cases} 0.04 - 0.15 w_{t-1} + 0.09 \Delta r_{Aaa,t-1} + 0.04 \Delta r_{Ibb,t-1} + u_{2t},\quad w_{t-1} \le 0.620, \\ (0.06)\ (0.15) \qquad (0.14) \qquad\qquad (0.10) \\ \\ -0.02 + 0.07 w_{t-1} - 0.18 \Delta r_{Aaa,t-1} + 0.21 \Delta r_{Ibb,t-1} + u_{2t},\ w_{t-1} > 0.620. \\ (0.12)\ (0.12) \qquad (0.15) \qquad\qquad (0.13) \end{cases} \tag{5.2.17}$$

For the pair of Ibb vs. Baa, with lag length equals to one and β estimated from the model, the estimated TVECM is given in equations (5.2.18) and (5.2.19):

$$\Delta r_{Baa,t} = \begin{cases} -0.00 - 0.00 w_{t-1} + 0.19 \Delta r_{Baa\,t-1} + 0.46 \Delta r_{Ibb,t-1} + u_{1t},\ w_{t-1} \le -0.529, \\ (0.02)\ (0.01) \qquad (0.05) \qquad\qquad (0.04) \\ \\ -0.07 - 0.19 w_{t-1} + 0.21 \Delta r_{Baa,t-1} - 0.03 \Delta r_{Ibb,t-1} + u_{1t},\ w_{t-1} > -0.529, \\ (0.02)\ (0.05) \qquad (0.12) \qquad\qquad (0.08) \end{cases} \tag{5.2.18}$$

and

$$\Delta r_{Ibb,t} = \begin{cases} 0.09 + 0.06 w_{t-1} - 0.08 \Delta r_{Baa t-1} + 0.24 \Delta r_{Ibb,t-1} + u_{2t}, \; w_{t-1} \leq -0.529, \\ (0.04)(0.03) \qquad (0.12) \qquad (0.09) \\ \\ -0.02 - 0.09 w_{t-1} - 0.18 \Delta r_{Baa,t-1} - 0.44 \Delta r_{Ibb,t-1} + u_{2t}, \; w_{t-1} > -0.529. \\ (0.03)\,(0.09) \qquad (0.24) \qquad (0.15) \end{cases} \tag{5.2.19}$$

For the pair of Ibb vs. Baa, with lag length equals to one, the estimated TVECM with fixed cointegrating vector is given in equations (5.2.20) and (5.2.21):

$$\Delta r_{Baa,t} = \begin{cases} 0.01 - 0.00 w_{t-1} + 0.25 \Delta r_{Baa,t-1} + 0.45 \Delta r_{Ibb,t-1} + u_{1t}, \; w_{t-1} \leq 1.920, \\ (0.02)\,(0.02) \qquad (0.06) \qquad (0.05) \\ \\ 0.02 - 0.03 w_{t-1} + 0.10 \Delta r_{Baa,t-1} + 0.34 \Delta r_{Ibb,t-1} + u_{1t}, \; w_{t-1} > 1.920, \\ (0.15)\,(0.06) \qquad (0.09) \qquad (0.06) \end{cases} \tag{5.2.20}$$

and

$$\Delta r_{Ibb,t} = \begin{cases} -0.01 + 0.00 w_{t-1} + 0.04 \Delta r_{Baa,t-1} + 0.17 \Delta r_{Ibb,t-1} + u_{2t}, \; w_{t-1} \leq 1.920, \\ (0.05)\,(0.04) \qquad (0.10) \qquad (0.08) \\ \\ -0.03 + 0.01 w_{t-1} - 0.31 \Delta r_{Baa,t-1} + 0.08 \Delta r_{Ibb,t-1} + u_{2t}, \; w_{t-1} > 1.920. \\ (0.29)\,(0.12) \qquad (0.23) \qquad (0.14) \end{cases} \tag{5.2.21}$$

5.2.5. Conclusions

The Hansen-Seo two-regime threshold vector error correction model was employed to estimate and test threshold behavior of four interest rates pairs in this section. The following conclusions were made:

- From the calculated p-values, Tsy vs. Aaa pair has strong threshold cointegration relationship with different lag length specifications, regardless of whether cointegrating parameter β is estimated from the model or is fixed at unity. For the pair Tsy vs. Baa, the p-values also show strong tendency that these two rates are threshold cointegrated with both lag specifications, when the cointegrating parameter β is estimated from the model. However, when β equals to unity, the calculated p-values deteriorate when we extend the lag length from one to two. For other two pairs of interest rates (Ibb vs. Aaa and Ibb vs. Baa), relatively high p-values were obtained under both lag length specifications, regardless of whether β is estimated or fixed at unity.

- When the cointegrating parameter β is estimated from the model, for both lag length specifications, six out of eight β estimates are greater than unity while two are less than unity. This finding is in contrast to the conventional assumption that the corporate bond rate will move one unit when the Treasury note is shocked by a unit shock. In particular, the estimated β are 1.385 and 1.377, for Ibb vs. Baa pair; while the estimated β are 0.981 and 0.948 for Ibb vs. Aaa pair, for 1-lag and 2-lag length.
- Whether the first regime or the second regime is the typical regime, depends on which interest pairs are of interest and how we select the lag length. For example, for the Ibb vs. Aaa pair when β is estimated from the model, under different lag length specification, the first regime will switch from typical regime to extreme regime.
- Two sensitivity tests were conducted in this section. One test was to extend the grid points search from 100 grid points to 300 grid points in a two-dimensional setting. The second test was to increase the bootstrap simulation replications from 1000 to 5000. Some gains were observed from changing the grid points from 100 to 300, since the AIC criterion will improve under a 300 points environment. A mixed

picture was obtained when increasing the simulation replications from 1000 to 5000. In some cases the p-values decreased, but in other cases, the p-values increased. However, the changes of the p-values were relatively moderate.

Table 5.2.1. Results of the threshold cointegration

(A). $\beta=1$

Lag = 1	Tsy vs. Aaa	Tsy vs. Baa	Ibb vs. Aaa	Ibb vs. Baa
AIC	-1634	-1594	-1454	-1463
BIC	-1623	-1584	-1442	-1453
1st regime obs*	20	36	65	74
2nd regime obs*	80	64	35	26
p-value**	0.013	0.047	0.502	0.990
p-value***	0.009	0.054	0.519	0.988
p-value****	0.017	0.083	0.517	0.961
p-value*****	0.020	0.069	0.511	0.974
Gamma hat	0.290	1.410	0.620	1.920

Lag = 2	Tsy vs. Aaa	Tsy vs. Baa	Ibb vs. Aaa	Ibb vs. Baa
AIC	-1644	-1586	-1445	-1456
BIC	-1628	-1570	-1430	-1440
1st regime obs*	20	36	65	77
2nd regime obs*	80	64	35	23
p-value**	0.078	0.138	0.470	0.633
p-value***	0.070	0.160	0.439	0.612
p-value****	0.099	0.211	0.475	0.639
p-value*****	0.101	0.208	0.487	0.609
Gamma hat	0.280	1.410	0.620	2.000

Note:
* In % term.
** Fixed regressor bootstrap with 1000 simulation replications.
*** Fixed regressor bootstrap with 5000 simulation replications.
**** Residual bootstrap with 1000 simulation replications.
***** Residual bootstrap with 5000 simulation replications.

Table 5.2.1. (continued)

(B). β is estimated from the model

Lag = 1	Tsy vs. Aaa	Tsy vs. Baa	Ibb vs. Aaa	Ibb vs. Baa
AIC	-1637	-1608	-1456	-1481
BIC	-1627	-1597	-1445	-1471
1st regime obs*	15	17	15	84
2nd regime obs*	85	83	85	16
p-value**	0.037	0.053	0.987	0.417
p-value***	0.039	0.054	0.987	0.422
p-value*****	0.051	0.061	0.956	0.380
p-value*****	0.044	0.064	0.954	0.380
Gamma hat	0.035	0.351	0.350	-0.529
Beta hat	1.039	1.108	0.981	1.385

Lag = 2	Tsy vs. Aaa	Tsy vs. Baa	Ibb vs. Aaa	Ibb vs. Baa
AIC	-1645	-1603	-1450	-147
BIC	-1629	-1587	-1434	-1450
1st regime obs*	16	16	79	85
2nd regime obs*	84	84	21	15
p-value**	0.059	0.038	0.953	0.268
p-value***	0.053	0.037	0.963	0.274
p-value****	0.069	0.049	0.910	0.277
p-value*****	0.075	0.056	0.903	0.283
Gamma hat	0.203	0.313	1.210	-0.465
Beta hat	1.007	1.112	0.948	1.377

Note:

*	In % term.
**	Fixed regressor bootstrap with 1000 simulation replications.
***	Fixed regressor bootstrap with 5000 simulation replications.
****	Residual bootstrap with 1000 simulation replications.
*****	Residual bootstrap with 5000 simulation replications.

Table 5.2.2. Results of the linear cointegration

(A). β=1

Lag = 1	Tsy vs. Aaa	Tsy vs. Baa	Ibb vs. Aaa	Ibb vs. Baa
AIC	949	1210	1496	1627
BIC	957	1218	1504	1635
test-statistic	-4.214	-3.694	-6.512	-4.050
Lag = 2				
AIC	937	1191	1493	1628
BIC	950	1203	1505	1640
test-statistic	-3.465	-2.960	-5.615	-3.747

* Test-statistic is the t-statistic of ρ in the Engle-Granger cointegration test: form the regression: $r_{b,t} = \beta_0 + \beta_1 r_{a,t} + \mu_t$, then form the autoregression of the residuals from the first step: $\Delta\hat{\mu}_{t-1} = \rho\hat{\mu}_{t-1} + \sum_{i=1}^{n} \rho_i \hat{\mu}_{t-i} + \varepsilon_t$. $\beta_1 = 1$ is restricted for the $\beta = 1$ panel.

** Critical values for 90%, 95% and 99% confidence intervals of the Augmented Dickey-Fuller Test:

Sample size	1%	5%	10%
100	-3.51	-2.89	-2.58
250	-3.46	-2.88	-2.57
500	-3.44	-2.87	-2.57

Table 5.2.2. (continued)

(B). β is estimated from the model

Lag = 1	Tsy vs. Aaa	Tsy vs. Baa	Ibb vs. Aaa	Ibb vs. Baa
AIC	952	1342	1507	1723
BIC	960	1351	1515	1732
test-statistic	-4.225	-5.200	-6.692	-5.814
Beta hat	1.002	1.135	1.018	1.151
Lag = 2				
AIC	940	1324	1504	1725
BIC	952	1336	1617	1737
test-statistic	-3.474	-4.081	-5.789	-5.419
Beta hat	1.002	1.135	1.018	1.151

* Test-statistic is the t-statistic of ρ in the Engle-Granger cointegration test: form the regression: $r_{b,t} = \beta_0 + \beta_1 r_{a,t} + \mu_t$, then form the autoregression of the residuals from the first step: $\Delta\hat{\mu}_{t-1} = \rho\hat{\mu}_{t-1} + \sum_{i=1}^{n} \rho_i \hat{\mu}_{t-i} + \varepsilon_t$. $\beta_1 = 1$ is restricted for the $\beta = 1$ panel.

** Critical values for 90%, 95% and 99% confidence intervals of the Augmented Dickey-Fuller Test:

Sample size	1%	5%	10%
100	-3.51	-2.89	-2.58
250	-3.46	-2.88	-2.57
500	-3.44	-2.87	-2.57

Table 5.2.3. Comparison of different grid search, Tsy vs. Aaa, $\beta = 1$

	Grid points=300		Grid points=100	
1st regime long rate eq	est. coeff.	std error	est. coeff.	std error
error-correction term	(0.1880)	0.3367	(0.1880)	0.3367
constant term	0.0798	0.0737	0.0798	0.0737
long rate difference lagged one term	(1.0823)	0.3659	(1.0823)	0.3659
short rate difference lagged one term	1.1029	0.2603	1.1029	0.2603
long rate difference lagged two term	(0.5433)	0.4850	(0.5433)	0.4850
short rate difference lagged two term	0.3777	0.3614	0.3777	0.3614
1st regime short rate eq				
error-correction term	0.2122	0.4586	0.2122	0.4586
constant term	(0.0122)	0.1019	(0.0122)	0.1019
long rate difference lagged one term	(1.0381)	0.5023	(1.0381)	0.5023
short rate difference lagged one term	1.0371	0.3894	1.0371	0.3894
long rate difference lagged two term	(0.4708)	0.7500	(0.4708)	0.7500
short rate difference lagged two term	0.1865	0.5490	0.1865	0.5490
2nd regime long rate eq				
error-correction term	(0.0570)	0.0340	(0.0570)	0.0340
constant term	0.0417	0.0300	0.0417	0.0300
long rate difference lagged one term	0.2159	0.1180	0.2159	0.1180
short rate difference lagged one term	0.2809	0.0762	0.2809	0.0762
long rate difference lagged two term	0.0080	0.1236	0.0080	0.1236
short rate difference lagged two term	(0.2371)	0.0972	(0.2371)	0.0972
2nd regime short rate eq				
error-correction term	(0.0018)	0.0467	(0.0018)	0.0467
constant term	0.0042	0.0412	0.0042	0.0412
long rate difference lagged one term	0.1310	0.1736	0.1310	0.1736
short rate difference lagged one term	0.3691	0.1143	0.3691	0.1143
long rate difference lagged two term	0.2424	0.1692	0.2424	0.1692
short rate difference lagged two term	(0.4084)	0.1262	(0.4084)	0.1262
AIC	-1644		-1644	
BIC	-1628		-1628	
1st regime % of total obs	20%		20%	
2nd regime % of total obs	80%		80%	
p-value: Fixed regressor bootstrap	0.078		0.094	
p-value: Residual bootstrap	0.099		0.141	
Gamma hat	0.280		0.280	

Table 5.2.4. Comparison of different grid search, Tsy vs. Aaa, β is estimated

	Grid points=300		Grid points=100	
1st regime long rate eq	est. coeff.	std error	est. coeff.	std error
error-correction term	(0.1654)	0.3117	(0.1679)	0.3033
constant term	0.0635	0.0590	0.0747	0.0633
long rate difference lagged one term	(1.2028)	0.3961	(1.1011)	0.3821
short rate difference lagged one term	1.1963	0.2709	1.1185	0.2541
long rate difference lagged two term	(0.6276)	0.4899	(0.5758)	0.4653
short rate difference lagged two term	0.4739	0.4011	0.4054	0.3621
1st regime short rate eq				
error-correction term	0.1674	0.4048	0.1981	0.3907
constant term	(0.0016)	0.0825	(0.0061)	0.0856
long rate difference lagged one term	(1.0973)	0.5505	(0.9911)	0.5189
short rate difference lagged one term	1.0892	0.4058	1.0060	0.3709
long rate difference lagged two term	(0.5661)	0.7513	(0.5225)	0.7157
short rate difference lagged two term	0.2873	0.6025	0.2200	0.5437
2nd regime long rate eq				
error-correction term	(0.0565)	0.0310	(0.0592)	0.0333
constant term	0.0388	0.0252	0.0430	0.0287
long rate difference lagged one term	0.2083	0.1169	0.2079	0.1177
short rate difference lagged one term	0.2878	0.0757	0.2873	0.0764
long rate difference lagged two term	0.0121	0.1238	0.0151	0.1245
short rate difference lagged two term	(0.2380)	0.0969	(0.2401)	0.0981
2nd regime short rate eq				
error-correction term	(0.0115)	0.0429	(0.0088)	0.0462
constant term	0.0139	0.0349	0.0113	0.0398
long rate difference lagged one term	0.1186	0.1728	0.1123	0.1736
short rate difference lagged one term	0.3785	0.1140	0.3827	0.1149
long rate difference lagged two term	0.2553	0.1696	0.2575	0.1705
short rate difference lagged two term	(0.4164)	0.1258	(0.4157)	0.1273
AIC	-1645		-1645	
BIC	-1629		-1629	
1st regime % of total obs	16%		19%	
2nd regime % of total obs	84%		81%	
p-value: Fixed regressor bootstrap	0.059		0.041	
p-value: Residual bootstrap	0.069		0.058	
Gamma hat	0.203		0.259	
Beta hat	1.007		1.002	

Table 5.2.5. Comparison of different grid search, Tsy vs. Baa, $\beta = 1$

	Grid points=300		Grid points=100	
1st regime long rate eq	est. coeff.	std error	est. coeff.	std error
error-correction term	0.0078	0.0239	0.0078	0.0239
constant term	0.0118	0.0221	0.0118	0.0221
long rate difference lagged one term	(0.0183)	0.0850	(0.0183)	0.0850
short rate difference lagged one term	0.5366	0.0701	0.5366	0.0701
1st regime short rate eq				
error-correction term	0.0561	0.0378	0.0561	0.0378
constant term	(0.0309)	0.0346	(0.0309)	0.0346
long rate difference lagged one term	(0.3006)	0.1657	(0.3006)	0.1657
short rate difference lagged one term	0.4207	0.1157	0.4207	0.1157
2nd regime long rate eq				
error-correction term	(0.0306)	0.0261	(0.0306)	0.0261
constant term	0.0500	0.0523	0.0500	0.0523
long rate difference lagged one term	0.1342	0.1218	0.1342	0.1218
short rate difference lagged one term	0.2557	0.0983	0.2557	0.0983
2nd regime short rate eq				
error-correction term	0.0280	0.0425	0.0280	0.0425
constant term	(0.0638)	0.0825	(0.0638)	0.0825
long rate difference lagged one term	(0.3220)	0.2207	(0.3220)	0.2207
short rate difference lagged one term	0.5355	0.1522	0.5355	0.1522
AIC	-1594		-1594	
BIC	-1584		-1584	
1st regime % of total obs	36%		36%	
2nd regime % of total obs	64%		64%	
p-value: Fixed regressor bootstrap	0.047		0.047	
p-value: Residual bootstrap	0.083		0.070	
Gamma hat	1.410		1.410	

Table 5.2.6. Comparison of different grid search, Tsy vs. Baa, β is estimated

	Grid points=300		Grid points=100	
1st regime long rate eq	est. coeff.	std error	est. coeff.	std error
error-correction term	(0.0918)	0.1370	(0.1197)	0.0904
constant term	0.0394	0.0240	0.0570	0.0256
long rate difference lagged one term	(0.5371)	0.1639	(0.4445)	0.1458
short rate difference lagged one term	0.9204	0.1570	0.8431	0.1312
1st regime short rate eq				
error-correction term	(0.0285)	0.2588	(0.1707)	0.1866
constant term	0.0230	0.0469	0.0622	0.0511
long rate difference lagged one term	(1.5300)	0.4002	(1.3135)	0.3608
short rate difference lagged one term	1.3159	0.3499	1.1368	0.2946
2nd regime long rate eq				
error-correction term	(0.0351)	0.0190	(0.0354)	0.0198
constant term	0.0344	0.0206	0.0384	0.0235
long rate difference lagged one term	0.2151	0.1051	0.2248	0.1088
short rate difference lagged one term	0.2160	0.0890	0.2097	0.0912
2nd regime short rate eq				
error-correction term	0.0177	0.0299	0.0126	0.0314
constant term	(0.0059)	0.0309	(0.0002)	0.0356
long rate difference lagged one term	(0.0422)	0.1621	(0.0355)	0.1676
short rate difference lagged one term	0.3889	0.1275	0.3833	0.1304
AIC	-1608		-1606	
BIC	-1597		-1596	
1st regime % of total obs	17%		20%	
2nd regime % of total obs	83%		80%	
p-value: Fixed regressor bootstrap	0.053		0.060	
p-value: Residual bootstrap	0.061		0.067	
Gamma hat	0.351		0.529	
Beta hat	1.108		1.095	

Table 5.2.7. Comparison of different grid search, Ibb vs. Aaa, $\beta = 1$

	Grid points=300		Grid points=100	
1st regime long rate eq	est. coeff.	std error	est. coeff.	std error
error-correction term	(0.2346)	0.0816	(0.2325)	0.0808
constant term	0.0928	0.0295	0.0923	0.0293
long rate difference lagged one term	0.0529	0.0776	0.0547	0.0771
short rate difference lagged one term	0.4265	0.0639	0.4253	0.0638
1st regime short rate eq				
error-correction term	(0.1463)	0.1468	(0.1506)	0.1458
constant term	0.0406	0.0564	0.0416	0.0562
long rate difference lagged one term	0.0946	0.1383	0.0909	0.1378
short rate difference lagged one term	0.0383	0.0989	0.0409	0.0988
2nd regime long rate eq				
error-correction term	(0.1144)	0.0755	(0.1141)	0.0763
constant term	0.1091	0.0738	0.1087	0.0747
long rate difference lagged one term	(0.0914)	0.0904	(0.0916)	0.0907
short rate difference lagged one term	0.5278	0.0706	0.5279	0.0706
2nd regime short rate eq				
error-correction term	0.0692	0.1206	0.0641	0.1218
constant term	(0.0210)	0.1189	(0.0152)	0.1204
long rate difference lagged one term	(0.1799)	0.1495	(0.1767)	0.1502
short rate difference lagged one term	0.2091	0.1274	0.2070	0.1273
AIC	-1454		-1453	
BIC	-1443		-1443	
1st regime % of total obs	65%		66%	
2nd regime % of total obs	35%		34%	
p-value: Fixed regressor bootstrap	0.502		0.541	
p-value: Residual bootstrap	0.517		0.580	
Gamma hat	0.620		0.630	

Table 5.2.8. Comparison of different grid search, Ibb vs. Aaa, β is estimated

	Grid points=300		Grid points=100	
1st regime long rate eq	est. coeff.	std error	est. coeff.	std error
error-correction term	0.7218	0.3076	0.7298	0.3040
constant term	(0.1981)	0.0903	(0.2057)	0.0911
long rate difference lagged one term	0.1289	0.2429	0.1249	0.2406
short rate difference lagged one term	0.8355	0.1902	0.8317	0.1887
1st regime short rate eq				
error-correction term	1.3466	0.5275	1.3288	0.5171
constant term	(0.3739)	0.1597	(0.3827)	0.1595
long rate difference lagged one term	0.4212	0.5144	0.4103	0.5160
short rate difference lagged one term	0.2650	0.3398	0.2642	0.3379
2nd regime long rate eq				
error-correction term	(0.0740)	0.0389	(0.0750)	0.0392
constant term	0.0527	0.0267	0.0544	0.0274
long rate difference lagged one term	(0.0243)	0.0637	(0.0242)	0.0638
short rate difference lagged one term	0.4591	0.0531	0.4585	0.0533
2nd regime short rate eq				
error-correction term	0.0524	0.0660	0.0487	0.0665
constant term	(0.0410)	0.0463	(0.0382)	0.0473
long rate difference lagged one term	(0.0659)	0.1068	(0.0656)	0.1068
short rate difference lagged one term	0.1330	0.0886	0.1307	0.0888
AIC	-1456		-1456	
BIC	-1445		-1445	
1st regime % of total obs	15%		15%	
2nd regime % of total obs	85%		85%	
p-value: Fixed regressor bootstrap	0.987		0.980	
p-value: Residual bootstrap	0.956		0.944	
Gamma hat	0.350		0.362	
Beta hat	0.981		0.980	

Table 5.2.9. Comparison of different grid search, Ibb vs. Baa, $\beta = 1$

	Grid points=300		Grid points=100	
1st regime long rate eq	est. coeff.	std error	est. coeff.	std error
error-correction term	(0.0006)	0.0193	0.0028	0.0178
constant term	0.0103	0.0216	0.0079	0.0206
long rate difference lagged one term	0.2497	0.0557	0.2563	0.0526
short rate difference lagged one term	0.4473	0.0493	0.4234	0.0457
1st regime short rate eq				
error-correction term	0.0036	0.0432	0.0350	0.0438
constant term	(0.0058)	0.0476	(0.0376)	0.0487
long rate difference lagged one term	0.0356	0.0999	0.0526	0.0957
short rate difference lagged one term	0.1700	0.0768	0.1294	0.0750
2nd regime long rate eq				
error-correction term	(0.0263)	0.0620	(0.0013)	0.0711
constant term	0.0191	0.1488	(0.0488)	0.1750
long rate difference lagged one term	0.1022	0.0899	0.1036	0.0926
short rate difference lagged one term	0.3351	0.0621	0.3515	0.0667
2nd regime short rate eq				
error-correction term	0.0050	0.1226	0.0804	0.1324
constant term	(0.0325)	0.2939	(0.2404)	0.3210
long rate difference lagged one term	(0.3118)	0.2327	(0.3326)	0.2442
short rate difference lagged one term	0.0817	0.1361	0.1297	0.1419
AIC	-1463		-1463	
BIC	-1453		-1452	
1st regime % of total obs	74%		78%	
2nd regime % of total obs	26%		22%	
p-value: Fixed regressor bootstrap	0.990		0.983	
p-value: Residual bootstrap	0.961		0.964	
Gamma hat	1.920		2.020	

Table 5.2.10. Comparison of different grid search, Ibb vs. Baa, β is estimated

	Grid points=300		Grid points=100	
1st regime long rate eq	est. coeff.	std error	est. coeff.	std error
error-correction term	(0.0009)	0.0119	(0.0022)	0.0122
constant term	(0.0013)	0.0169	(0.0045)	0.0185
long rate difference lagged one term	0.1943	0.0536	0.1863	0.0540
short rate difference lagged one term	0.4582	0.0350	0.4625	0.0361
1st regime short rate eq				
error-correction term	0.0620	0.0314	0.0671	0.0321
constant term	0.0898	0.0426	0.1069	0.0466
long rate difference lagged one term	(0.0799)	0.1197	(0.0664)	0.1214
short rate difference lagged one term	0.2385	0.0857	0.2185	0.0878
2nd regime long rate eq				
error-correction term	(0.1867)	0.0512	(0.1393)	0.0579
constant term	(0.0738)	0.0209	(0.0676)	0.0273
long rate difference lagged one term	0.2097	0.1150	0.2238	0.1233
short rate difference lagged one term	(0.0349)	0.0778	0.0705	0.1172
2nd regime short rate eq				
error-correction term	(0.0890)	0.0856	0.0346	0.1070
constant term	(0.0240)	0.0320	0.0089	0.0481
long rate difference lagged one term	(0.1794)	0.2448	(0.2700)	0.2429
short rate difference lagged one term	(0.4361)	0.1524	(0.1595)	0.2370
AIC	-1481		-1479	
BIC	-1471		-1468	
1st regime % of total obs	84%		81%	
2nd regime % of total obs	16%		19%	
p-value: Fixed regressor bootstrap	0.417		0.359	
p-value: Residual bootstrap	0.380		0.327	
Gamma hat	-0.529		-0.645	
Beta hat	1.385		1.393	

5.3. Results from the Enders-Siklos Threshold Cointegration Model

5.3.1. Introduction

The null of a linear unit root process against the alternative of the M-C TAR models for the various interest rate spreads was rejected in the data description section. This rejection provides strong evidence against linearity. According to the alternative hypothesis, the spreads are stationary and therefore the interest rate pairs are cointegrated with cointegrating vector [1, -1]. However, the alternative hypothesis of the spreads analysis imposes a very strong restriction on the values of cointegrating vector, i.e., that it is exactly equal to [1, -1]. It would be interesting to conduct a cointegrating test that does not restrict the cointegrating vector to be [1, -1]. The Enders-Siklos methodology relaxes the restriction on the values of cointegrating vector by estimating a long-run equilibrium relationship of interest rate pairs. Tables 5.3.1 to 5.3.4 report the test results for the Enders-Siklos momentum-consistent threshold autoregressive (M-C TAR) cointegration test[9].

In addition to reporting the results for the M-C TAR with consistent adjusted threshold, the results were also reported for the conventional Engle-Granger symmetric cointegration test (Engle-Granger), the Enders-Siklos test for threshold autoregression with the threshold equal to zero (TAR), the Enders-Siklos test for threshold autoregression with estimated threshold (C-TAR), and the Enders-Siklos test for momentum threshold autoregression with the threshold equal to zero (M-TAR).

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[9] The M-C TAR unit root tests introduced previously focused on whether the spreads of interest rate pairs have unit roots or are stationary. That is, they test the null that the interest rate pair are not cointegrated against the alternative that they are cointegrated with cointegrating vector [1, -1]. The testing for cointegration is without restricting the cointegrating vectors, *a priori*.

TAR and M-TAR are two nonlinear models assuming the threshold parameter is zero, so that the cointegrating vector coincides with the linear attractor. However, in general, the value of threshold parameter is unknown and needs to be estimated with other variables, e.g., ρ_1 and ρ_2. Referring to the estimation procedures of the methodology section, if the null hypothesis ($\rho_1 = \rho_2 = 0$) of nonstationary is rejected then it is possible to test for symmetric versus asymmetric adjustment (i.e. $\rho_1 = \rho_2$). In other words, if the null is rejected, so that the cointegrating residual sequence has an attractor then we can perform a standard F-test for symmetric adjustment. Tong (1983) demonstrated that if the adjustment process is asymmetric then the sample mean (i.e., under the assumption of zero threshold) is a biased estimate of the attractor. To rectify this bias, Chan (1993) showed that selecting a threshold candidate (from the cointegrating residual sequence) to minimize the sum of squared errors from the fitted model yields a super-consistent estimate of the threshold.

The reasons to compare these five models are: (1) one may compare the linear versus nonlinear adjustment; and (2) if there is no *a priori* information about the true threshold value then one may estimate the threshold through Chan's (1993) method instead of assuming the threshold is set at zero. If a cointegration relationship exists between an interest rate pair then the next question would be interesting to ask: Is the adjustment process to the long-term equilibrium symmetric or asymmetric? In other words, is the adjustment process state-dependent, displaying threshold behavior? From the methodology section, it is recalled that the TAR model can capture the "deep" cyclical process documented by Sichel (1993), while the M-TAR model can detect the "sharp" movements documented in DeLong and Summer (1986) and Sichel (1993).

5.3.2. Discussion

The test results for the equality of ρ_1 and ρ_2 (Tables 5.3.1 to 5.3.4) show that asymmetric adjustment holds for three out of four cases at the 5% significance level under the M-C TAR model. The exception is the Baa-Ibb pair, which does, however, display significant asymmetry under the TAR model with a consistent estimator of the threshold (C-TAR). For example, consider the pair for Aaa versus Tsy. Table 5.3.2 shows that asymmetric adjustment is most evident for the M-C TAR with consistent adjusted threshold. Specifically, the p-value for the test of equality of ρ_1 and ρ_2 for the M-C TAR model (0.008) is much smaller than the p-values for the zero threshold TAR (0.609), M-TAR (0.097), and C-TAR (0.021) tests. In addition, note that the AIC and BIC model selection criteria indicate that the M-C TAR performs better than the linear cointegration model fit as part of the Engle-Granger test.

Overall, the following three conclusions can be made:

First, as shown in Tables 5.3.1 to 5.3.4, at the 5% significance level, all tests reject the symmetry null in favor of the M-C TAR (Aaa vs. Ibb, Aaa vs. Tsy, and Baa vs. Tsy) and C-TAR models (Baa vs. Ibb and Aaa vs. Tsy).

Second, the results can be compared between the M-C TAR and the M-TAR with threshold set to zero. The M-C TAR provides much stronger evidence of asymmetric behavior than the M-TAR with zero threshold. Consider the case of Aaa vs. Tsy (Table 5.3.2). The F-test ($\rho_1 = \rho_2$) statistic is 7.004 with a significance level of 0.008 under the M-C TAR model, but it is only 2.773 with a 0.097 significance level under the M-TAR. Hence, at the 10% significance level, both models suggest the existence of a cointegration relationship ($\phi = 9.618$, and 7.445, respectively). However, at the 5% level, symmetry is rejected against the M-C TAR, but it is not rejected against the M-TAR with zero threshold. Thus, the asymmetric adjustment evidence is greatly enhanced under the M-C TAR with adjusted threshold regime. One may draw a similar conclusion when comparing the C-TAR model versus the TAR model for any of the other three interest rate pairs.

Third, the AIC and BIC model selection criteria can be the basis of further conclusions. Note, in contrast to the R-square measure, the AIC and BIC model selection criteria incorporate penalties for additional parameters. Thus, although the M-C TAR model nests the M-TAR and linear cointegration models, the AIC and BIC should be smallest for the M-C TAR model, only if the additional parameters substantially improve the model's fit. Consider, for example, the Aaa vs. Ibb case in Table 5.3.1. The AIC of the M-C TAR model is 1504, which is slightly better than the AIC of the linear cointegration, i.e., Engle-Granger model (1507), the TAR model with zero threshold (1505), the M-TAR model with zero threshold (1505), and the C-TAR model (1508). Similar conclusions can be drawn from the Aaa vs. Tsy case in Table 5.3.2. However, for the pairs Baa vs. Ibb and Baa vs. Tsy, the AIC and BIC statistics under the TAR model with zero threshold are the lowest across the five models.

5.3.3. Error-correction models with asymmetric adjustments

Consider the case of Aaa vs. Tsy. The analysis conducted in the previous section indicated that this pair provides the strongest evidence against symmetric adjustment in the sense that the null of no cointegration is rejected against the nonlinear cointegration alternative at the smallest test size. Next, with the confirmation of a cointegration relationship, the error-correction model was estimated, which incorporates the momentum consistent threshold autoregressive asymmetric adjustment structure (with t-statistics in parentheses)[10]:

$$\Delta r_{Tsy,t} = \underset{(0.198)}{0.0026} - \underset{(-0.063)}{0.0028} I_t \hat{\mu}_{t-1} - \underset{(-0.138)}{0.0099}(1-I_t)\hat{\mu}_{t-1} + \underset{(4.443)}{0.4717}\Delta r_{Tsy,t-1} - \underset{(-3.160)}{0.3390}\Delta r_{Tsy,t-2}$$

$$- \underset{(-0.588)}{0.0831}\Delta r_{Aaa,t-1} + \underset{(1.086)}{0.1417}\Delta r_{Aaa,t-2}, \qquad (5.3.1)$$

and

$$\Delta r_{Aaa,t} = \underset{(0.452)}{0.0044} - \underset{(-1.226)}{0.0391} I_t \hat{\mu}_{t-1} - \underset{(-2.714)}{0.1431}(1-I_t)\hat{\mu}_{t-1} + \underset{(5.172)}{0.4030}\Delta r_{Tsy,t-1} - \underset{(-2.101)}{0.1654}\Delta r_{Tsy,t-2}$$

$$- \underset{(-0.028)}{0.0029}\Delta r_{Aaa,t-1} - \underset{(-0.563)}{0.0539}\Delta r_{Aaa,t-2}, \qquad (5.3.2)$$

where: $\hat{\mu}_{t-1} = r_{Aaa,t-1} - 0.6693 - 1.0019 r_{Tsy,t-1}$, and the Heaviside indicator is given by:

$$I_t = \begin{cases} 1, & \text{if } \Delta\mu_{t-1} \geq -0.080, \\ 0, & \text{if } \Delta\mu_{t-1} < -0.080. \end{cases} \qquad (5.3.3)$$

Notice that, $\hat{\mu}_{t-1} = r_{Aaa,t-1} - 0.6693 - 1.0019 r_{Tsy,t-1}$, is the residual of the estimated long-run equilibrium relationship. Following the Enders-Siklos methodology, the estimated long-run equilibrium relationship (with t statistics in parentheses) is:

$$r_{Aaa,t-1} = \underset{(12.340)}{0.6693} + \underset{(145.742)}{1.0019} r_{Tsy,t-1} + \hat{\mu}_{t-1},$$

where r_{Aaa} and r_{Tsy} are the values of Aaa and Tsy, respectively.

The above estimates show that both Aaa and Tsy adjust moderately to positive change in $\hat{\mu}_{t-1}$. Within any month, the Tsy rate adjusts roughly 0.28% and the Aaa rate

[10] In the error correction setting, lagged terms were estimated to be from zero to three. Then, based on AIC and BIC criteria, the best fitted model is reported. The same procedures apply to other pairs.

adjusts 3.91% in response to a positive 1-unit deviation from the long-run equilibrium. For a negative 1-unit deviation from the long-run equilibrium, the corporate rates Aaa adjust more strongly (14.31%) than the Tsy rates (about 0.99%). Thus, adjustment to the long-run equilibrium shows a strong tendency when the Aaa rates wander away from the cointegration relationship with a negative change in $\hat{\mu}_{t-1}$.

For comparison purposes, the linear error-correction model was also estimated with symmetric adjustment. Continuing to look at the case of Tsy vs. Aaa, one can determine:

$$\begin{aligned} \Delta r_{Tsy,t} &= \underset{(0.200)}{0.0027} - \underset{(-0.122)}{0.0046\hat{\mu}_{t-1}} + \underset{(4.447)}{0.4714\Delta r_{Tsy,t-1}} - \underset{(-3.168)}{0.3393\Delta r_{Tsy,t-2}} \\ &\quad - \underset{(-0.584)}{0.0824\Delta r_{Aaa,t-1}} + \underset{(1.086)}{0.1414\Delta r_{Aaa,t-2}}, \end{aligned} \tag{5.3.4}$$

and

$$\begin{aligned} \Delta r_{Aaa,t} &= \underset{(0.480)}{0.0047} - \underset{(-2.367)}{0.0660\hat{\mu}_{t-1}} + \underset{(5.103)}{0.3983\Delta r_{Tsy,t-1}} - \underset{(-2.153)}{0.1698\Delta r_{Tsy,t-2}} \\ &\quad + \underset{(0.076)}{0.0078\Delta r_{Aaa,t-1}} - \underset{(-0.595)}{0.0571\Delta r_{Aaa,t-2}}, \end{aligned} \tag{5.3.5}$$

where: $\hat{\mu}_{t-1} = r_{Aaa,t-1} - 0.6693 - 1.0019 r_{Tsy,t-1}$.

Within the regime of symmetric adjustment, these estimates show that for both rates the adjustments to the long-run equilibrium are relatively slow. Overall, for a 1-unit gap away from long-run equilibrium, Tsy and Aaa adjust roughly 0.46% and 6.60%, respectively, regardless of the sign of the deviation from long-run equilibrium.

For the pair of Aaa vs. Ibb, under the asymmetric adjustment regime (M-C TAR), the error-correction model is given in equations (5.3.6) and (5.3.7):

$$\Delta r_{Ibb,t} = \underset{(0.460)}{0.0070} + \underset{(0.706)}{0.0387} I_t \hat{\mu}_{t-1} + \underset{(1.608)}{0.1873}(1-I_t)\hat{\mu}_{t-1} + \underset{(1.515)}{0.1178}\Delta r_{Ibb,t-1} + \underset{(0.236)}{0.0198}\Delta r_{Ibb,t-2}$$

$$+ \underset{(0.193)}{0.0238}\Delta r_{Aaa,t-1} - \underset{(-2.569)}{0.1953}\Delta r_{Aaa,t-2} \tag{5.3.6}$$

and

$$\Delta r_{Aaa,t} = \underset{(0.435)}{0.0036} - \underset{(-2.391)}{0.0719} I_t \hat{\mu}_{t-1} - \underset{(-1.211)}{0.0773}(1-I_t)\hat{\mu}_{t-1} + \underset{(10.785)}{0.4593}\Delta r_{Ibb,t-1} + \underset{(0.193)}{0.0089}\Delta r_{Ibb,t-2}$$

$$+ \underset{(0.521)}{0.0352}\Delta r_{Aaa,t-1} - \underset{(-3.286)}{0.1368}\Delta r_{Aaa,t-2} \tag{5.3.7}$$

where: $\hat{\mu}_{t-1} = r_{Aaa,t-1} - 0.4260 - 1.0178 r_{Ibb,t-1}$, and the Heaviside indicator is given by:

$$I_t = \begin{cases} 1, & \text{if } \Delta\mu_{t-1} \geq -0.218, \\ 0, & \text{if } \Delta\mu_{t-1} < -0.218. \end{cases} \tag{5.3.8}$$

Notice that the estimated long-run equilibrium relationship (with t statistics in parentheses) is:

$$r_{Aaa,t-1} = \underset{(8.281)}{0.4260} + \underset{(158.157)}{1.0178} r_{Ibb,t-1} + \hat{\mu}_{t-1},$$

where r_{Aaa} and r_{Ibb} are the values of Aaa and Ibb, respectively.

Similarly, the linear error-correction model with symmetric adjustment is:

$$\Delta r_{Ibb,t} = \underset{(0.302)}{0.0045} + \underset{(1.131)}{0.0593}\hat{\mu}_{t-1} + \underset{(1.461)}{0.1135}\Delta r_{Ibb,t-1} + \underset{(0.234)}{0.0197}\Delta r_{Ibb,t-2},$$

$$+ \underset{(0.171)}{0.0212}\Delta r_{Aaa,t-1} - \underset{(-2.522)}{0.1917}\Delta r_{Aaa,t-2}, \tag{5.3.9}$$

and

$$\Delta r_{Aaa,t} = 0.0037 - 0.0722\hat{\mu}_{t-1} + 0.4607\Delta r_{Ibb,t-1} + 0.0093\Delta r_{Ibb,t-2}$$
$$(0.450)\ (-2.521)\qquad (10.852)\qquad (0.202)$$

$$+0.0338\Delta r_{Aaa,t-1} - 0.1365\Delta r_{Aaa,t-2}, \qquad (5.3.10)$$
$$(0.499)\qquad (-3.286)$$

with: $\hat{\mu}_{t-1} = r_{Aaa,t-1} - 0.4260 - 1.0178 r_{Ibb,t-1}$.

For the pair of Baa vs. Ibb, under the asymmetric adjustment regime (C-TAR), the error-correction model is given in equations (5.3.11) and (5.3.12):

$$\Delta r_{Ibb,t} = 0.0018 + 0.0488 I_t\hat{\mu}_{t-1} + 0.0306(1 - I_t)\hat{\mu}_{t-1} + 0.1503\Delta r_{Ibb,t-1} - 0.0920\Delta r_{Baa,t-1},$$
$$(0.107)\ (1.368)\qquad (0.482)\qquad (2.609)\qquad (-1.156)$$

$$(5.3.11)$$

and

$$\Delta r_{Baa,t} = -0.0078 - 0.0122 I_t\hat{\mu}_{t-1} - 0.1090(1 - I_t)\hat{\mu}_{t-1} + 0.3747\Delta r_{Ibb,t-1} + 0.1848\Delta r_{Baa,t-1}$$
$$(-0.987)\ (-0.722)\qquad (-3.636)\qquad (13.774)\qquad (4.923)$$

$$(5.3.12)$$

where: $\hat{\mu}_{t-1} = r_{Baa,t-1} - 0.4186 - 1.1514 r_{Ibb,t-1}$, and the Heaviside indicator is given by:

$$I_t = \begin{cases} 1, & \text{if } \mu_{t-1} \geq -0.478, \\ 0, & \text{if } \mu_{t-1} < -0.478. \end{cases} \qquad (5.3.13)$$

Notice that the estimated long-run equilibrium relationship (with t statistics in parentheses) is:

$$r_{Baa,t-1} = 0.4186 + 1.1514 r_{Ibb,t-1} + \hat{\mu}_{t-1},$$
$$(5.239)\ (115.217)$$

where r_{Baa} and r_{Ibb} are the values of Baa and Ibb, respectively.

Similarly, the linear error-correction model with symmetric adjustment is:

$$\Delta r_{Ibb,t} = \underset{(0.241)}{0.0036} + \underset{(1.414)}{0.04491}\hat{\mu}_{t-1} + \underset{(2.655)}{0.1522}\Delta r_{Ibb,t-1} - \underset{(-1.148)}{0.0910}\Delta r_{Baa,t-1}, \tag{5.3.14}$$

and

$$\Delta r_{Baa,t} = \underset{(0.285)}{0.0020} - \underset{(-2.260)}{0.0342}\hat{\mu}_{t-1} + \underset{(13.976)}{0.3816}\Delta r_{Ibb,t-1} + \underset{(5.082)}{0.1918}\Delta r_{Baa,t-1}, \tag{5.3.15}$$

where: $\hat{\mu}_{t-1} = r_{Baa,t-1} - 0.4186 - 1.1514 r_{Ibb,t-1}$.

For the pair of Baa vs. Tsy, under the asymmetric adjustment regime (M-C TAR), the error-correction model is given in equations (5.3.16) and (5.3.17):

$$\Delta r_{Tsy,t} = \underset{(0.203)}{0.0027} - \underset{(-0.114)}{0.0035}I_t\hat{\mu}_{t-1} + \underset{(0.413)}{0.0237}(1-I_t)\hat{\mu}_{t-1} + \underset{(5.478)}{0.4571}\Delta r_{Tsy,t-1} - \underset{(-3.653)}{0.2878}\Delta r_{Tsy,t-2}$$

$$- \underset{(-0.583)}{0.0760}\Delta r_{Baa,t-1} + \underset{(0.859)}{0.0975}\Delta r_{Baa,t-2}, \tag{5.3.16}$$

and

$$\Delta r_{Baa,t} = \underset{(0.373)}{0.0032} - \underset{(-1.907)}{0.0369}I_t\hat{\mu}_{t-1} - \underset{(-2.638)}{0.0962}(1-I_t)\hat{\mu}_{t-1} + \underset{(5.232)}{0.2775}\Delta r_{Tsy,t-1} - \underset{(-2.808)}{0.1406}\Delta r_{Tsy,t-2}$$

$$+ \underset{(2.462)}{0.2041}\Delta r_{Baa,t-1} + \underset{(0.616)}{0.0445}\Delta r_{Baa,t-2}, \tag{5.3.17}$$

where: $\hat{\mu}_{t-1} = r_{Baa,t-1} - 0.6810 - 1.1352 r_{Tsy,t-1}$, and the Heaviside indicator is given by:

$$I_t = \begin{cases} 1, & \text{if } \Delta\mu_{t-1} \geq -0.151, \\ 0, & \text{if } \Delta\mu_{t-1} < -0.151. \end{cases} \tag{5.3.18}$$

Notice that the estimated long-run equilibrium relationship (with t statistics in parentheses) is:

$$r_{Baa,t-1} = \underset{(8.742)}{0.6810} + \underset{(114.965)}{1.1352} r_{Tsy,t-1} + \hat{\mu}_{t-1},$$

where r_{Baa} and r_{Tsy} are the values of Baa and Tsy, respectively.

Similarly, the linear error-correction model with symmetric adjustment is given in equations (5.3.19) and (5.3.20):

$$\begin{aligned}\Delta r_{Tsy,t} &= \underset{(0.207)}{0.0028} + \underset{(0.065)}{0.0018}\hat{\mu}_{t-1} + \underset{(5.536)}{0.4601}\Delta r_{Tsy,t-1} - \underset{(-3.632)}{0.2847}\Delta r_{Tsy,t-2} \\ &\quad - \underset{(-0.639)}{0.0827}\Delta r_{Baa,t-1} + \underset{(0.841)}{0.0952}\Delta r_{Baa,t-2},\end{aligned} \tag{5.3.19}$$

and

$$\begin{aligned}\Delta r_{Baa,t} &= \underset{(0.358)}{0.0030} - \underset{(-2.723)}{0.0484}\hat{\mu}_{t-1} + \underset{(5.120)}{0.2711}\Delta r_{Tsy,t-1} - \underset{(-2.951)}{0.1474}\Delta r_{Tsy,t-2} \\ &\quad + \underset{(2.653)}{0.2187}\Delta r_{Baa,t-1} + \underset{(0.685)}{0.0494}\Delta r_{Baa,t-2},\end{aligned} \tag{5.3.20}$$

where: $\hat{\mu}_{t-1} = r_{Baa,t-1} - 0.6810 - 1.1352 r_{Tsy,t-1}$.

5.4. Recommendations

With the new methodologies to obtain deeper understanding of the adjustment structure, one may conduct further analysis to investigate whether there are gains by incorporating the asymmetric adjustment structure into model building.

Table 5.3.1. Cointegration test for Aaa vs. Ibb (sample: 1/1960 to 12/1997, n = 456)

Model (Lag = 1)	Engle-Granger	TAR	M-TAR	C-TAR (a_0 = -0.301)	M-C TAR (a_0 = -0.218)
ρ_1	-0.240	-0.234	-0.259	-0.221	-0.212
	(-6.693)[e]	(-5.313)	(-5.379)	(-5.400)	(-5.553)
ρ_2	NA	-0.251	-0.220	-0.296	-0.404
		(-4.475)[f]	(-4.365)	(-4.469)	(-4.726)
γ_1	-0.215	-0.215	-0.211	-0.214	-0.209
	(-4.674)[g]	(-4.669)	(-4.532)	(-4.664)	(-4.571)
AIC/BIC[a]	1507/1515	1505/1518	1505/1517	1508/1522	1504/1517
Φ[b]	NA	22.376	22.532	22.885	24.788
$\rho_1=\rho_2$[c]	NA	0.057	0.341	0.989	4.446
		(0.812)	(0.560)	(0.321)	(0.036)
Q(4)[d]	4.602	4.615	4.682	4.766	3.862
	(0.331)[h]	(0.329)	(0.321)	(0.312)	(0.425)

Note:

[a] AIC = T*ln(residual sum of squares) + 2*n; BIC = T*ln((residual sum of squares) + n*ln(T), where n = number of regressors and T = number of usable observations.

[b] Entries in this row are the sample F-statistics for testing the null of $\rho_1 = \rho_2 = 0$.

[c] Entries in this row are the sample F-statistics for the null hypothesis that adjustments are symmetric. The corresponding significance levels are contained in brackets.

[d] Q(4) is the Ljung-Box statistics for the joint hypotheses of no serial correlation among the first four residuals.

[e] Entries in the brackets of this row are the t-statistics for the null hypothesis $\rho_1 = 0$.

[f] Entries in the brackets of this row are the t-statistics for the null hypothesis $\rho_2 = 0$.

[g] Entries in the brackets of this row are the t-statistics for the null hypothesis $\gamma_1 = 0$.

[h] Entries in the brackets of this row are the significance level for the Ljung-Box Q-statistics.

Table 5.3.2. Cointegration test for Aaa vs. Tsy (sample: 1/1960 to 12/1997, n = 456)

Model (Lag = 2)	Engle-Granger	TAR	M-TAR	C-TAR (a_0 = -0.443)	M-C TAR (a_0 = -0.080)
ρ_1	-0.059	-0.051	-0.026	-0.035	-0.034
	(-3.474)[e]	(-2.219)	(-0.998)	(-1.732)	(-1.732)
ρ_2	NA	-0.068	-0.082	-0.118	-0.134
		(-2.784)[f]	(-3.753)	(-3.868)	(-4.070)
γ_1	-0.060	0.060	0.052	0.062	0.059
	(-1.292)[g]	(1.289)	(1.129)	(1.349)	(1.292)
γ_2	-0.167	-0.168	-0.176	-0.167	-0.171
	(-3.619)[g]	(-3.633)	(-3.788)	(-3.624)	(-3.727)
AIC/BIC[a]	940/952	937/954	939/956	936/953	935/951
Φ[b]	NA	6.156	7.445	8.785	9.618
$\rho_1 = \rho_2$[c]	NA	0.261	2.773	5.381	7.004
		(0.609)	(0.097)	(0.021)	(0.008)
Q(4)[d]	1.442	1.285	1.577	1.309	1.880
	(0.837)[h]	(0.864)	(0.813)	(0.860)	(0.758)

Note:

[a] AIC = T*ln(residual sum of squares) + 2*n; BIC = T*ln((residual sum of squares) + n*ln(T), where n = number of regressors and T = number of usable observations.

[b] Entries in this row are the sample F-statistics for testing the null of $\rho_1 = \rho_2 = 0$.

[c] Entries in this row are the sample F-statistics for the null hypothesis that adjustments are symmetric. The corresponding significance levels are contained in brackets.

[d] Q(4) is the Ljung-Box statistics for the joint hypotheses of no serial correlation among the first four residuals.

[e] Entries in the brackets of this row are the t-statistics for the null hypothesis $\rho_1 = 0$.

[f] Entries in the brackets of this row are the t-statistics for the null hypothesis $\rho_2 = 0$.

[g] Entries in the brackets of this row are the t-statistics for the null hypothesis $\gamma_1 = 0$.

[h] Entries in the brackets of this row are the significance level for the Ljung-Box Q-statistics.

Table 5.3.3. Cointegration test for Baa vs. Ibb (sample: 1/1960 to 12/1997, n = 456)

Model (Lag = 3)	Engle-Granger	TAR	M-TAR	C-TAR (a_0 = -0.478)	M-C TAR (a_0 = -0.071)
ρ_1	-0.133	-0.117	-0.149	-0.102	-0.101
	(-4.439)[e]	(-3.200)	(-3.494)	(-3.040)	(-2.706)
ρ_2	NA	-0.160	-0.120	-0.225	-0.182
		(-3.506)[f]	(-3.112)	(-4.111)	(-4.054)
γ_1	-0.140	-0.140	-0.135	-0.136	-0.149
	(-2.874)[g]	(-2.864)	(-2.704)	(-2.796)	(-3.038)
γ_2	-0.075	-0.075	-0.074	-0.075	-0.078
	(-1.560)[g]	(-1.565)	(-1.521)	(-1.563)	(-1.611)
γ_3	-0.165	-0.166	-0.164	-0.169	-0.167
	(-3.538)[g]	(-3.560)	(-3.504)	(-3.634)	(-3.593)
AIC/BIC[a]	1714/1730	1708/1728	1715/1736	1712/1732	1714/1734
Φ[b]	NA	10.152	9.972	11.942	10.959
$\rho_1=\rho_2$[c]	NA	0.617	0.272	4.023	2.162
		(0.433)	(0.602)	(0.045)	(0.142)
Q(4)[d]	1.333	1.385	1.330	1.331	1.302
	(0.856)[h]	(0.847)	(0.856)	(0.856)	(0.861)

Note:
[a] AIC = T*ln(residual sum of squares) + 2*n; BIC = T*ln((residual sum of squares) + n*ln(T), where n = number of regressors and T = number of usable observations.
[b] Entries in this row are the sample F-statistics for testing the null of $\rho_1 = \rho_2 = 0$.
[c] Entries in this row are the sample F-statistics for the null hypothesis that adjustments are symmetric. The corresponding significance levels are contained in brackets.
[d] Q(4) is the Ljung-Box statistics for the joint hypotheses of no serial correlation among the first four residuals.
[e] Entries in the brackets of this row are the t-statistics for the null hypothesis $\rho_1 = 0$.
[f] Entries in the brackets of this row are the t-statistics for the null hypothesis $\rho_2 = 0$.
[g] Entries in the brackets of this row are the t-statistics for the null hypothesis $\gamma_1 = 0$.
[h] Entries in the brackets of this row are the significance level for the Ljung-Box Q-statistics.

Table 5.3.4. Cointegration test for Baa vs. Tsy (sample: 1/1960 to 12/1997, n = 456)

Model (Lag = 2)	Engle-Granger	TAR	M-TAR	C-TAR (a_0 = -0.569)	M-C TAR (a_0 = -0.151)
ρ_1	-0.074	-0.061	-0.044	-0.053	-0.053
	(-4.081)[e]	(-2.590)	(-1.667)	(-2.462)	(-2.605)
ρ_2	NA	-0.093	-0.101	-0.123	-0.156
		(-3.396)[f]	(-4.114)	(-3.906)	(-4.006)
γ_1	0.231	0.232	0.224	0.232	0.225
	(5.103)[g]	(5.111)	(4.910)	(5.134)	(4.977)
γ_2	-0.208	-0.209	-0.218	-0.209	-0.214
	(-4.521)[g]	(-4.552)	(-4.711)	(-4.558)	(-4.671)
AIC/BIC[a]	1324/1336	1319/1336	1323/1340	1322/1339	1320/1337
Φ [b]	NA	8.747	9.642	10.269	11.221
$\rho_1 = \rho_2$ [c]	NA	0.842	2.569	3.778	5.614
		(0.359)	(0.110)	(0.053)	(0.018)
Q(4)[d]	6.312	6.358	5.739	6.745	5.600
	(0.177)[h]	(0.174)	(0.220)	(0.150)	(0.231)

Note:

[a] AIC = T*ln(residual sum of squares) + 2*n; BIC = T*ln((residual sum of squares) + n*ln(T), where n = number of regressors and T = number of usable observations.

[b] Entries in this row are the sample F-statistics for testing the null of $\rho_1 = \rho_2 = 0$.

[c] Entries in this row are the sample F-statistics for the null hypothesis that adjustments are symmetric. The corresponding significance levels are contained in brackets.

[d] Q(4) is the Ljung-Box statistics for the joint hypotheses of no serial correlation among the first four residuals.

[e] Entries in the brackets of this row are the t-statistics for the null hypothesis $\rho_1 = 0$.

[f] Entries in the brackets of this row are the t-statistics for the null hypothesis $\rho_2 = 0$.

[g] Entries in the brackets of this row are the t-statistics for the null hypothesis $\gamma_1 = 0$.

[h] Entries in the brackets of this row are the significance level for the Ljung-Box Q-statistics.

CHAPTER 6. FORECASTING PERFORMANCE EVALUATION

6.1. Introduction

After obtaining different ways of modeling corporate bond and Treasury rates, it is important to determine which models offer better forecasts, since good forecasts will usually lead to better decision-making. To evaluate which model provides the best forecasting performance, one-step-ahead to six-step-ahead forecasts of six estimated models were applied as described in the previous sections using six forecast accuracy measures: (a) mean error (ME); (b) error variance (EV); (c) root mean squared error (RMSE); (d) root mean squared percent error (RMSPE); (e) mean absolute error (MAE); and (f) mean absolute percent error (MAPE). The following models were evaluated: (1) Lo-Zivot three-regime models with lag length equal to 1 and 2 (equations 5.1.2-5.1.17); (2) Hansen-Seo two-regime model with cointegration vector estimated from the model ("β estimated" in the table); and (3) Hansen-Seo two-regime model with cointegration vector set to unity ("β = 1" in the table, equations 5.2.2 to 5.2.5 and 5.2.10 to 5.2.21); (4) Enders-Siklos two-regime model (equations 5.3.1-5.3.2, 5.3.6-5.3.7, 5.3.11-5.3.12, and 5.3.16-5.3.17); and (5) Engle-Granger linear model (equations 5.3.4-5.3.5, 5.3.9-5.3.10, 5.3.14-5.3.15, and 5.3.19-5.3.20). In addition, the Neal-Rolph-Morris (2000, hereafter NRM) Johansen type cointegration model was also included in the comparison for the pairs of Aaa vs. Tsy and Baa vs. Tsy (equations 6.1-6.4). The results were extracted from Table 6 of NRM, with the following cointegrated vector error-correction models. For the Aaa vs. Tsy pair (t-statistics are shown at parentheses):

$$\begin{array}{l} \Delta r_{Tsy,t} = -0.0007 + 0.0069(r_{Aaa,t-1} - 1.028 r_{Tsy,t-1}) + 0.4762 \Delta r_{Tsy,t-1} - 0.3370 \Delta r_{Tsy,t-2} \\ \qquad\quad (-0.030) \quad (0.185) \qquad\qquad\qquad\qquad\qquad (4.496) \qquad\quad (-3.146) \end{array}$$

$$\underset{(-0.601)}{-0.0847\Delta r_{Aaa,t-1}} \underset{(1.094)}{+0.1425\Delta r_{Aaa,t-2}}, \tag{6.1}$$

and

$$\Delta r_{Aaa,t} = \underset{(1.974)}{0.0324} \underset{(-2.123)}{-0.0585}(r_{Aaa,t-1} - 1.028 r_{Tsy,t-1}) \underset{(5.143)}{+0.4017\Delta r_{Tsy,t-1}} \underset{(-2.130)}{-0.1682\Delta r_{Tsy,t-2}}$$

$$\underset{(0.052)}{+0.0054\Delta r_{Aaa,t-1}} \underset{(-0.602)}{-0.0579\Delta r_{Aaa,t-2}}. \tag{6.2}$$

For the Baa vs. Tsy pair:

$$\Delta r_{Tsy,t} = \underset{(-0.096)}{-0.0016} \underset{(0.432)}{+0.0119}(r_{Baa,t-1} - 1.178 r_{Tsy,t-1}) \underset{(5.597)}{+0.4647\Delta r_{Tsy,t-1}} \underset{(-3.610)}{-0.2828\Delta r_{Tsy,t-2}}$$

$$\underset{(-0.626)}{-0.0809\Delta r_{Baa,t-1}} \underset{(0.883)}{+0.1002\Delta r_{Baa,t-2}}, \tag{6.3}$$

and

$$\Delta r_{Baa,t} = \underset{(1.747)}{0.0186} \underset{(-2.469)}{-0.0433}(r_{Baa,t-1} - 1.178 r_{Tsy,t-1}) \underset{(5.167)}{+0.2738\Delta r_{Tsy,t-1}} \underset{(-2.918)}{-0.1458\Delta r_{Tsy,t-2}}$$

$$\underset{(2.637)}{+0.2177\Delta r_{Baa,t-1}} \underset{(0.680)}{+0.0492\Delta r_{Baa,t-2}}. \tag{6.4}$$

6.2. Forecasting methodology

First, the estimated models were used to forecast based on the period (from 1/1998 through 12/2002), and then the forecasts were compared to the actual data. In each h-step-ahead forecast (h = 1, 2, …, 6), a bivariate time series was generated by incorporating the coefficients estimated from the fitted model of each pair of rates. Sixty periods of long rates

(Aaa, or Baa) and short rates (Tsy, or Ibb) were generated in each system of equations using the following procedure: (a) using data through 12/1997, make a one-step-ahead forecast (and compute the forecast error); (b) using the estimated model and data through 1/1998, make another one-step-ahead forecast (and compute the forecast error); (c) using the data through 2/1998, and so on. This provides a sequence of one-step-ahead forecast errors for each estimated model. One could then compare, for example, the root mean-squared one-step-ahead forecast errors across models. The same method can be applied for two-step-ahead forecasts: utilize data through 12/1997 to generate a two-step-ahead forecast by incorporating the one-step-ahead forecast value obtained from previous step as the value of 1/1998. Then, use data through 1/1998 to generate the next two-step-ahead forecast, and so on. This sequence is repeated for three, four, five, and six-step-ahead forecasts across models.

After determining the forecast values, six common accuracy measures were calculated to evaluate the forecast performance: mean error (ME), error variance (EV), root mean squared error (RMSE), root mean squared percent error (RMSPE), mean absolute error (MAE), and mean absolute percent error (MAPE). Because ME measures bias, other things being the same, a forecast with a small ME was preferred. EV measures dispersion of the forecast errors. Other things being the same, a forecast with small EV was preferred. Some other accuracy measures that were performed are: RMSE, RMSPE, MAE and MAPE (see Appendix C for detailed definitions).

6.3. Forecasting performance evaluation

Tables 6.3.1 to 6.3.14 report the results of the forecast performance of long rates (Aaa and Baa) versus short rate (Treasury). In Tables 6.3.15 to 6.3.26, the forecasted short rate is compared to the Ibbotson bond index. In each table, six measures of forecast accuracy are calculated for one-step-ahead forecast to six-step-ahead forecast for both long and short rates. The lower panel also calculates the sum of each measure for both long and short rates.

Tables 6.3.1 to 6.3.7 consider the pair of Aaa vs. Tsy. The second column of the table presents the result of 1-step-ahead forecast. The accuracy measure ME shows that, on average, all seven models under-estimate the values of Aaa around 1.23 to 5.40 bp (note that: 1 bp = 1 basis point = 0.01%), since ME > 0. In contrast, on average, six forecast models consistently over-estimate the values of Tsy in the range of 2.43 to 23.84 bp for 1-step-ahead forecasts, since ME < 0. The exception is the Lo-Zivot three-regime model with lag length set to two, on average, this model underestimates the Tsy around 40.40 bp. The results also show that all forecast accuracy measures follow the correct pattern, i.e., as h increases, the magnitude of forecast accuracy measure increases.

Across different model specifications, if one focuses on the result of 1-step-ahead forecast, the specification of Enders-Siklos' M-C TAR, with lag length equal to two, offers the best performance with four smallest accuracy measures: RMSE, RMPSE, MAE and MAPE, among the seven models. The three-regime Lo-Zivot specification (with one lag in level) offers the smallest EV. For the 6-step-ahead forecast, the EV and RMSE are smallest in the specification of NRM. The other linear model, Engle-Granger, has the smallest RMSPE and MAE among the seven models. However, Hansen-Seo, with unity cointegrating vector, provides the smallest MAPE. It is interesting to observe that the threshold cointegration model such as Enders-Siklos dictates the 1-step-ahead forecast performance, whereas two linear cointegration models such as Engle-Granger, and NRM have better 6-step-ahead forecast performance. Figures 6.1 to 6.3 depict the out-of-sample forecasting values versus the actual rates of Enders-Siklos, Engle-Granger and NRM models.

Tables 6.3.8 to 6.3.14 summarize the forecasting performance results of Baa vs. Tsy pair. For the Baa forecasting, the 1-step-ahead forecasting results show that, on average, all seven models under-estimate the actual Baa rates in the range from 2.66 bp to 5.44 bp, since ME > 0. Five of the seven models over-estimate the Tsy rates in the range from 1.83 bp to 4.59 bp with the exception of two Lo-Zivot three-regime models. Lo-Zivot models underestimate the Tsy rates from 1.08 bp to 10.59 bp. Moreover, Hansen-Seo two-regime, with unity cointegrating vector, offers the best fit among seven models: for both 1-step-ahead and 6-step-ahead forecasts, the accuracy measures are smallest among models (except the EV of 1-step-ahead forecast). For this model, on average, 1-step-ahead forecast of the Baa rates are underestimated by about 3.60 bp, while the forecast of the Treasury rates are overestimated by about 1.83 bp. However, the 6-step-ahead forecast error is larger. On average, the Baa rates are underestimated by 15.69 bp, while the Treasury rates are overestimated by 21.38 bp. This model tackles the rate spreads directly by setting the cointegrating vector as [1, -1], against estimating the cointegrating vector from the model. The other model of Hansen-Seo, with cointegrating vector estimated from the model, is the runner-up for the 1-step-ahead forecasting performance with five better accuracy measures: EV, RMSE, RMPSE, MAE and MAPE. For comparison purposes, figures of the out-of-sample forecasted values of two Hansen-Seo two-regime threshold cointegration models and the linear NRM model are presented in Figures 6.4 to 6.6.

The forecasting performance evaluation of Aaa vs. Ibb pair is summarized in Tables 6.3.15 to 6.3.20. For the 1-step-ahead forecast, on average, all six models underestimate the Aaa rates (in the range from 1.77 bp to 14.21 bp). Similarly, four out of six models overestimate the Ibb rates (in the range of 3.82 bp to 7.06 bp) with the exception of two Lo-Zivot three-regime models (underestimates 22.48 bp and 9.16 bp, respectively). The previous exceptions show a similar pattern as the Baa vs. Tsy pair. That is, long rates are under-estimated for all specifications, whereas short rates are over-estimated for five models, with the exception of two Lo-Zivot three-regime specifications. Hansen-Seo two-regime, with estimated cointegrating vector, offers the best fit. All five accuracy measures (EV, RMSE, RMSPE, MAE and MAPE) are the smallest among the six models for both 1-step-ahead and 6-step-ahead forecasts. In addition, it must be noted that, in this Hansen-Seo model, the Aaa rates are underestimated about 3.69 bp (1-step-ahead) to 7.54 bp (6-step-ahead), while the Ibb rates are overestimated about 3.82 bp (1-step-ahead) to 20.13 bp (6-step-ahead). For comparison purpose, the figures of two Hansen-Seo two-regime threshold cointegration models are presented in Figures 6.7 and 6.8.

For the pair of Baa vs. Ibb, as shown from Tables 6.3.21 to 6.3.26, on average, for the 1-step-ahead forecast, five out of six models underestimate the Baa rates (in the range of 1.06 bp to 5.07 bp) with the exception of Lo-Zivot three-regime model, with lag length equal to one, (overestimates by 1.30 bp). In contrast, four out of six models overestimate the Ibb rates (0.72 bp to 4.89 bp) with the exception of two Lo-Zivot three-regime models (underestimates by 3.07 bp and 0.31 bp, respectively). For 1-step-ahead forecast evaluation, Lo-Zivot three-regime with lag equal to one offers the best fit for five accuracy measures (EV, RMSE, RMPSE, MAE and MAPE). For 6-step-ahead forecast evaluation, Hansen-Seo two-regime with unity cointegrating vector exhibits the best fit. The forecasted values and the actual rates of two better-fitted models are presented in Figures 6.9 and 6.10.

6.4. Lo-Zivot Three-Regime Model with a non-unity cointegrating vector

In the estimations of the Lo-Zivot three-regime threshold cointegration models, the cointegrating vector were set to [1, -1], because there is no prior information about the values of the cointegrating vectors. The results of other model specifications, e.g., NRM and Hansen-Seo two-regime models, have confirmed that it is not necessary for the cointegrating vector to be [1, -1]. Hence, further investigation was conducted. The Lo-Zivot three-regime threshold cointegration model was estimated with lag length set to two in levels by incorporating different cointegrating vectors obtained from other model specifications. For example, for the pair of Aaa versus Tsy, two more specifications were estimated by incorporating the cointegrating vectors of [1, -1.028] (from NRM) and [1, -1.039] (from Hansen-Seo). For the pair of Baa versus Tsy, the cointegrating vectors of [1, -1.178] (from NRM) and [1, -1.108] (from Hansen-Seo) were incorporated. For the pairs of Aaa vs. Ibb and Baa vs. Ibb, the cointegrating vectors are [1, -0.981] and [1, -1.385], respectively. The estimated coefficients and key results of aforementioned six models are displayed in Table 6.4.1. Tables 6.4.2 to 6.4.7 summarize the 1-step-ahead to 6-step-ahead forecasting performance evaluation.

One may compare the results summarized by Table 6.4.1 (with non-unity cointegrating vector) to the results from Table 5.2.1 (with unity cointegrating vector). For the Aaa vs. Tsy pair, with the cointegrating vectors set to [1, -1.028], [1, -1.039], and [1, -1], the distribution of the observations fell into different regimes are as the following: (68, 96, 290), (70, 71, 313), and (121, 131, 202). The $LR_{1,3}$ statistics increase from 43.89 to 52.09 and 67.34, if one relaxes the unity cointegrating vector assumption. The p-values decrease from 0.022 to 0.004 and 0.001 with 1000 simulation replications. For the pair of Baa vs. Tsy, with the cointegrating vectors set to [1, -1.178], [1, -1.108], and [1, -1], the distribution of the observations fell into different regimes are: (75, 271, 108), (78, 309, 67), and (234, 115, 105). With non-unity cointegrating vectors, over half of the observations fell into the middle random walk regimes versus 26% for the unity cointegrating vector specification.

For the Aaa vs. Ibb pair, with the cointegrating vector [1, -0.981], the calculated p-value was initially 0.011. However, it was 0.869 when the cointegrating vector was set to [1, -1]. A similar pattern was observed for the pair of Baa vs. Ibb. The calculated p-value approaches zero if the cointegrating vector is [1, -1.385], while the calculated p-value was 0.567 with unity cointegrating vector.

The following conclusions were made from the out-of-sample forecast evaluation of above models with non-unity cointegrating vectors. In general, there are still some gains. For the 1-step-ahead forecast, the conclusions of the leading forecasting model in each pair of interest rates stand the same. However, for the 6-step-ahead forecast, for the Baa vs. Tsy pair, if one sets the cointegrating vector as [1, -1.1781] then this specification offers better accuracy measures (RMSE, RMSPE, MAE, MAPE) than the Hansen-Seo specification with unity cointegrating vector.

6.5. Conclusions

A few conclusions can be drawn from the forecasting performance evaluation:

- For the 1-step-ahead forecast evaluation, on average, most of the forecasting models show a pattern of underestimating the corporate bond indices (Aaa and Baa), but overestimating the Treasury note and Ibbotson bond index with a few exceptions.
- In general, for the 1-step-ahead forecast, the forecasting accuracy measures show that the threshold cointegration models perform better than their linear cointegration model counterparts. The specification of Enders-Siklos M-C TVECM offers the best forecasting performance to the pair of Aaa versus Tsy. The Hansen-Seo two-regime threshold cointegration with unity cointegrating vector specification provides the best forecasting performance of the pair Baa versus Tsy. For the pair of Aaa versus Ibb, the Hansen-Seo two-regime threshold cointegration, with estimated cointegrating vector specification, presents the best forecasting performance. The Lo-Zivot three-regime threshold cointegration model, with lag length set to one in levels, is the best forecasting model for the pair of Baa versus Ibb.
- The threshold cointegration specifications perform better than the linear cointegration models, but none of the threshold cointegration models dictate the overall performance across different pairs of rates.
- Non-unity estimated cointegrating vectors were incorporated from other model specifications into Lo-Zivot three-regime model. Forecasting performance evaluation results have shown that there are some gains for this inclusion.

Table 6.3.1. Forecasting performance (Aaa vs. Tsy), Lo-Zivot, lag = 1

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0357	0.0712	0.1061	0.1306	0.1567	0.1836
EV	0.0224	0.0448	0.0716	0.0991	0.1367	0.1731
RMSE	0.1538	0.2232	0.2879	0.3408	0.4016	0.4548
RMSPE	0.0217	0.0317	0.0407	0.0479	0.0557	0.0624
MAE	0.1154	0.1718	0.2195	0.2461	0.2888	0.3161
MAPE	0.0165	0.0246	0.0312	0.0348	0.0407	0.0441
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.2384)	(0.3921)	(0.5423)	(0.5642)	(0.5540)	(0.5215)
EV	0.0611	0.1418	0.2392	0.3270	0.3851	0.4289
RMSE	0.3433	0.5437	0.7302	0.8033	0.8318	0.8372
RMSPE	0.0709	0.1147	0.1557	0.1726	0.1781	0.1779
MAE	0.2878	0.4347	0.5918	0.6426	0.6789	0.7015
MAPE	0.0571	0.0879	0.1204	0.1315	0.1384	0.1418
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.2027)	(0.3209)	(0.4362)	(0.4336)	(0.3972)	(0.3379)
EV	0.0834	0.1866	0.3108	0.4261	0.5218	0.6020
RMSE	0.4971	0.7669	1.0181	1.1442	1.2334	1.2920
RMSPE	0.0926	0.1464	0.1964	0.2205	0.2338	0.2402
MAE	0.4032	0.6066	0.8113	0.8886	0.9678	1.0175
MAPE	0.0736	0.1124	0.1516	0.1663	0.1790	0.1860

Table 6.3.2. Forecasting performance (Aaa vs. Tsy), Lo-Zivot, lag = 2

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0123	0.1565	0.3964	0.6901	1.1188	1.5034
EV	0.0301	0.0570	0.0905	0.1472	0.2971	0.4146
RMSE	0.1740	0.2854	0.4976	0.7896	1.2445	1.6355
RMSPE	0.0246	0.0407	0.0713	0.1132	0.1773	0.2306
MAE	0.1351	0.2243	0.4153	0.6947	1.1234	1.5080
MAPE	0.0193	0.0321	0.0594	0.0995	0.1605	0.2145
Tsy	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.4040	1.1379	2.0034	3.2163	4.2579	4.7526
EV	0.0911	0.3954	0.9397	1.7388	2.0103	1.9388
RMSE	0.5043	1.3001	2.2256	3.4761	4.4877	4.9524
RMSPE	0.1002	0.2595	0.4422	0.6772	0.8539	0.9288
MAE	0.4226	1.1414	2.0060	3.2188	4.2604	4.7552
MAPE	0.0823	0.2220	0.3893	0.6198	0.8107	0.8985
Sum	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.4164	1.2944	2.3998	3.9064	5.3766	6.2560
EV	0.1212	0.4523	1.0302	1.8860	2.3074	2.3534
RMSE	0.6783	1.5855	2.7232	4.2657	5.7322	6.5879
RMSPE	0.1248	0.3002	0.5135	0.7904	1.0312	1.1593
MAE	0.5577	1.3656	2.4213	3.9136	5.3838	6.2632
MAPE	0.1016	0.2541	0.4487	0.7193	0.9712	1.1130

Table 6.3.3. Forecasting performance (Aaa vs. Tsy), Hansen-Seo, lag = 2, β estimated

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0441	0.0874	0.1326	0.1743	0.2146	0.2516
EV	0.0373	0.0657	0.0903	0.1104	0.1459	0.1915
RMSE	0.1981	0.2708	0.3285	0.3751	0.4381	0.5048
RMSPE	0.0279	0.0384	0.0463	0.0527	0.0610	0.0695
MAE	0.1562	0.2133	0.2491	0.2858	0.3272	0.3620
MAPE	0.0222	0.0306	0.0353	0.0404	0.0461	0.0507
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0243)	(0.0517)	(0.0598)	(0.0649)	(0.0687)	(0.0707)
EV	0.0598	0.1391	0.2072	0.2739	0.3559	0.4207
RMSE	0.2458	0.3765	0.4591	0.5274	0.6005	0.6525
RMSPE	0.0465	0.0746	0.0924	0.1074	0.1216	0.1305
MAE	0.2051	0.2942	0.3752	0.4506	0.5031	0.5416
MAPE	0.0389	0.0574	0.0733	0.0885	0.0988	0.1053
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0198	0.0356	0.0729	0.1093	0.1459	0.1809
EV	0.0971	0.2048	0.2975	0.3842	0.5018	0.6123
RMSE	0.4439	0.6473	0.7875	0.9025	1.0386	1.1573
RMSPE	0.0744	0.1130	0.1387	0.1601	0.1826	0.2000
MAE	0.3613	0.5075	0.6243	0.7364	0.8304	0.9036
MAPE	0.0611	0.0879	0.1087	0.1289	0.1449	0.1560

Table 6.3.4. Forecasting performance (Aaa vs. Tsy), Hansen-Seo, lag = 2, $\beta = 1$

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0441	0.0859	0.1302	0.1713	0.2109	0.2468
EV	0.0372	0.0654	0.0901	0.1101	0.1453	0.1904
RMSE	0.1978	0.2698	0.3272	0.3735	0.4356	0.5013
RMSPE	0.0278	0.0383	0.0461	0.0525	0.0606	0.0690
MAE	0.1560	0.2128	0.2482	0.2843	0.3250	0.3584
MAPE	0.0222	0.0305	0.0352	0.0402	0.0458	0.0502
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0299)	(0.0636)	(0.0757)	(0.0839)	(0.0903)	(0.0949)
EV	0.0596	0.1384	0.2056	0.2709	0.3508	0.4134
RMSE	0.2460	0.3774	0.4597	0.5272	0.5991	0.6499
RMSPE	0.0467	0.0750	0.0929	0.1079	0.1220	0.1307
MAE	0.2057	0.2953	0.3765	0.4481	0.4995	0.5359
MAPE	0.0390	0.0577	0.0737	0.0882	0.0984	0.1044
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0141	0.0224	0.0545	0.0874	0.1206	0.1519
EV	0.0968	0.2038	0.2957	0.3811	0.4960	0.6038
RMSE	0.4438	0.6472	0.7869	0.9007	1.0347	1.1512
RMSPE	0.0745	0.1133	0.1390	0.1603	0.1826	0.1997
MAE	0.3617	0.5081	0.6247	0.7324	0.8245	0.8943
MAPE	0.0612	0.0882	0.1089	0.1284	0.1441	0.1546

Table 6.3.5. Forecasting performance (Aaa vs. Tsy), Enders-Siklos, M-C TAR, lag = 2

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0463	0.0762	0.1007	0.1227	0.1464	0.1686
EV	0.0327	0.0567	0.0833	0.1237	0.1615	0.2108
RMSE	0.1867	0.2501	0.3057	0.3724	0.4277	0.4891
RMSPE	0.0263	0.0355	0.0433	0.0526	0.0599	0.0676
MAE	0.1475	0.1983	0.2329	0.2852	0.3220	0.3474
MAPE	0.0210	0.0284	0.0332	0.0406	0.0457	0.0490
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0251)	(0.0647)	(0.0959)	(0.1191)	(0.1404)	(0.1577)
EV	0.0579	0.1309	0.1947	0.2637	0.3421	0.4044
RMSE	0.2419	0.3675	0.4516	0.5271	0.6015	0.6552
RMSPE	0.0458	0.0733	0.0922	0.1091	0.1240	0.1336
MAE	0.1992	0.2919	0.3683	0.4416	0.4947	0.5486
MAPE	0.0377	0.0569	0.0724	0.0874	0.0980	0.1075
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0211	0.0115	0.0048	0.0036	0.0061	0.0110
EV	0.0906	0.1876	0.2780	0.3873	0.5036	0.6152
RMSE	0.4285	0.6176	0.7573	0.8996	1.0292	1.1442
RMSPE	0.0721	0.1087	0.1355	0.1618	0.1839	0.2011
MAE	0.3467	0.4902	0.6012	0.7268	0.8167	0.8960
MAPE	0.0587	0.0853	0.1056	0.1280	0.1437	0.1565

Table 6.3.6. Forecasting performance (Aaa vs. Tsy), Engle-Granger, lag = 2

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0540	0.0943	0.1240	0.1511	0.1796	0.2061
EV	0.0372	0.0619	0.0807	0.1012	0.1381	0.1833
RMSE	0.2002	0.2660	0.3100	0.3522	0.4127	0.4751
RMSPE	0.0282	0.0378	0.0437	0.0494	0.0574	0.0654
MAE	0.1545	0.2098	0.2381	0.2718	0.3073	0.3384
MAPE	0.0220	0.0301	0.0338	0.0385	0.0433	0.0475
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0246)	(0.0640)	(0.0943)	(0.1194)	(0.1398)	(0.1565)
EV	0.0580	0.1308	0.1943	0.2617	0.3405	0.4023
RMSE	0.2420	0.3673	0.4508	0.5253	0.6000	0.6533
RMSPE	0.0458	0.0732	0.0919	0.1086	0.1236	0.1331
MAE	0.1993	0.2917	0.3670	0.4429	0.4957	0.5481
MAPE	0.0377	0.0568	0.0721	0.0875	0.0981	0.1074
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0294	0.0304	0.0297	0.0317	0.0398	0.0496
EV	0.0951	0.1927	0.2750	0.3629	0.4785	0.5856
RMSE	0.4423	0.6333	0.7608	0.8775	1.0127	1.1285
RMSPE	0.0740	0.1110	0.1356	0.1581	0.1809	0.1985
MAE	0.3538	0.5015	0.6051	0.7147	0.8029	0.8865
MAPE	0.0597	0.0869	0.1059	0.1260	0.1414	0.1548

Table 6.3.7. Forecasting performance (Aaa vs. Tsy), Neal-Rolph-Morris, lag = 2

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0503	0.0826	0.1049	0.1268	0.1509	0.1729
EV	0.0373	0.0622	0.0812	0.1013	0.1373	0.1817
RMSE	0.1995	0.2627	0.3036	0.3426	0.4001	0.4600
RMSPE	0.0281	0.0374	0.0428	0.0482	0.0557	0.0634
MAE	0.1544	0.2087	0.2340	0.2632	0.2988	0.3249
MAPE	0.0220	0.0299	0.0332	0.0373	0.0422	0.0456
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0356)	(0.0897)	(0.1318)	(0.1660)	(0.1945)	(0.2188)
EV	0.0579	0.1303	0.1929	0.2588	0.3358	0.3959
RMSE	0.2432	0.3719	0.4585	0.5352	0.6113	0.6661
RMSPE	0.0462	0.0746	0.0943	0.1118	0.1273	0.1375
MAE	0.2007	0.2955	0.3730	0.4443	0.4960	0.5656
MAPE	0.0381	0.0578	0.0735	0.0882	0.0988	0.1115
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0147	(0.0070)	(0.0268)	(0.0392)	(0.0436)	(0.0459)
EV	0.0951	0.1925	0.2740	0.3602	0.4732	0.5776
RMSE	0.4427	0.6346	0.7621	0.8778	1.0114	1.1261
RMSPE	0.0743	0.1119	0.1371	0.1599	0.1830	0.2008
MAE	0.3551	0.5042	0.6069	0.7075	0.7948	0.8904
MAPE	0.0600	0.0878	0.1068	0.1256	0.1409	0.1571

Table 6.3.8. Forecasting performance (Baa vs. Tsy), Lo-Zivot, lag = 1

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0266	0.0534	0.0765	0.0752	0.0793	0.1101
EV	0.0436	0.0945	0.1670	0.2552	0.3856	0.4783
RMSE	0.2106	0.3120	0.4157	0.5108	0.6260	0.7003
RMSPE	0.0267	0.0405	0.0543	0.0671	0.0821	0.0911
MAE	0.1687	0.2660	0.3562	0.4361	0.5278	0.6020
MAPE	0.0215	0.0343	0.0461	0.0567	0.0685	0.0777
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0108	0.0210	0.0263	0.0259	0.0347	0.0572
EV	0.0628	0.1601	0.2622	0.3874	0.5241	0.6280
RMSE	0.2509	0.4007	0.5127	0.6230	0.7248	0.7945
RMSPE	0.0480	0.0784	0.1004	0.1212	0.1394	0.1504
MAE	0.1989	0.3259	0.4428	0.5485	0.6239	0.6801
MAPE	0.0380	0.0628	0.0852	0.1052	0.1190	0.1284
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0374	0.0743	0.1028	0.1011	0.1140	0.1672
EV	0.1065	0.2546	0.4292	0.6427	0.9097	1.1063
RMSE	0.4615	0.7127	0.9285	1.1337	1.3508	1.4948
RMSPE	0.0747	0.1189	0.1547	0.1883	0.2215	0.2415
MAE	0.3677	0.5919	0.7990	0.9847	1.1517	1.2821
MAPE	0.0595	0.0971	0.1314	0.1618	0.1875	0.2062

Table 6.3.9. Forecasting performance (Baa vs. Tsy), Lo-Zivot, lag = 2

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0321	0.0912	0.1696	0.2563	0.3710	0.5058
EV	0.0374	0.0656	0.0914	0.1217	0.1957	0.3324
RMSE	0.1960	0.2719	0.3466	0.4329	0.5774	0.7669
RMSPE	0.0244	0.0341	0.0440	0.0551	0.0730	0.0964
MAE	0.1465	0.2063	0.2832	0.3432	0.4473	0.5984
MAPE	0.0185	0.0261	0.0360	0.0438	0.0566	0.0754
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.1059	0.2694	0.4418	0.6448	0.8861	1.1595
EV	0.0743	0.2304	0.4396	0.7732	1.3092	2.0154
RMSE	0.2924	0.5505	0.7967	1.0904	1.4472	1.8330
RMSPE	0.0571	0.1109	0.1594	0.2135	0.2798	0.3528
MAE	0.2260	0.4317	0.6444	0.9066	1.2166	1.5547
MAPE	0.0432	0.0842	0.1254	0.1751	0.2339	0.2988
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.1380	0.3606	0.6114	0.9011	1.2571	1.6652
EV	0.1117	0.2961	0.5310	0.8950	1.5049	2.3478
RMSE	0.4884	0.8224	1.1433	1.5233	2.0246	2.5999
RMSPE	0.0815	0.1450	0.2034	0.2686	0.3528	0.4492
MAE	0.3725	0.6380	0.9275	1.2498	1.6639	2.1531
MAPE	0.0617	0.1103	0.1614	0.2189	0.2906	0.3742

Table 6.3.10. Forecasting performance (Baa vs. Tsy), Hansen-Seo, lag = 1, β estimated

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0411	0.0803	0.1132	0.1280	0.1405	0.1564
EV	0.0380	0.0651	0.0772	0.0870	0.1103	0.1448
RMSE	0.1993	0.2675	0.3001	0.3216	0.3606	0.4115
RMSPE	0.0248	0.0336	0.0378	0.0407	0.0454	0.0513
MAE	0.1507	0.2082	0.2452	0.2649	0.2770	0.3050
MAPE	0.0190	0.0264	0.0311	0.0337	0.0350	0.0384
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0459)	(0.1070)	(0.1740)	(0.2446)	(0.3060)	(0.3570)
EV	0.0538	0.1232	0.1839	0.2533	0.3388	0.4015
RMSE	0.2365	0.3670	0.4628	0.5596	0.6576	0.7272
RMSPE	0.0451	0.0734	0.0958	0.1190	0.1406	0.1553
MAE	0.1970	0.2965	0.3727	0.4459	0.5284	0.6044
MAPE	0.0375	0.0580	0.0738	0.0897	0.1067	0.1217
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0047)	(0.0267)	(0.0608)	(0.1166)	(0.1655)	(0.2005)
EV	0.0919	0.1883	0.2611	0.3404	0.4491	0.5463
RMSE	0.4359	0.6345	0.7628	0.8812	1.0182	1.1387
RMSPE	0.0698	0.1069	0.1336	0.1597	0.1860	0.2066
MAE	0.3477	0.5048	0.6179	0.7109	0.8054	0.9094
MAPE	0.0566	0.0843	0.1050	0.1234	0.1417	0.1601

Table 6.3.11. Forecasting performance (Baa vs. Tsy), Hansen-Seo, lag = 1, β = 1

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0360	0.0735	0.1044	0.1200	0.1362	0.1569
EV	0.0375	0.0620	0.0717	0.0796	0.1023	0.1336
RMSE	0.1969	0.2595	0.2873	0.3065	0.3476	0.3978
RMSPE	0.0245	0.0326	0.0362	0.0389	0.0438	0.0497
MAE	0.1479	0.1999	0.2338	0.2539	0.2682	0.3042
MAPE	0.0187	0.0253	0.0297	0.0323	0.0339	0.0383
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0183)	(0.0534)	(0.1011)	(0.1499)	(0.1868)	(0.2138)
EV	0.0548	0.1247	0.1879	0.2608	0.3457	0.4089
RMSE	0.2348	0.3571	0.4451	0.5322	0.6169	0.6742
RMSPE	0.0449	0.0711	0.0918	0.1127	0.1311	0.1429
MAE	0.1912	0.2977	0.3653	0.4292	0.4876	0.5543
MAPE	0.0366	0.0580	0.0722	0.0861	0.0982	0.1108
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0177	0.0201	0.0033	(0.0298)	(0.0506)	(0.0569)
EV	0.0923	0.1866	0.2596	0.3404	0.4479	0.5425
RMSE	0.4318	0.6166	0.7325	0.8388	0.9645	1.0721
RMSPE	0.0693	0.1036	0.1280	0.1516	0.1749	0.1925
MAE	0.3391	0.4976	0.5991	0.6832	0.7558	0.8584
MAPE	0.0552	0.0833	0.1019	0.1184	0.1321	0.1491

Table 6.3.12. Forecasting performance (Baa vs. Tsy), Enders-Siklos, M-C TAR, lag = 2

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0539	0.1040	0.1547	0.1936	0.2314	0.2706
EV	0.0408	0.0676	0.0839	0.1062	0.1283	0.1681
RMSE	0.2091	0.2800	0.3283	0.3790	0.4265	0.4913
RMSPE	0.0259	0.0351	0.0411	0.0475	0.0531	0.0607
MAE	0.1582	0.2192	0.2628	0.3049	0.3265	0.3599
MAPE	0.0199	0.0277	0.0332	0.0385	0.0410	0.0450
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0300)	(0.0731)	(0.1112)	(0.1448)	(0.1709)	(0.1917)
EV	0.0572	0.1282	0.1886	0.2534	0.3360	0.3997
RMSE	0.2410	0.3655	0.4483	0.5238	0.6043	0.6606
RMSPE	0.0458	0.0729	0.0916	0.1089	0.1254	0.1364
MAE	0.1989	0.2901	0.3624	0.4351	0.4905	0.5504
MAPE	0.0378	0.0566	0.0713	0.0862	0.0974	0.1084
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0239	0.0309	0.0435	0.0488	0.0605	0.0789
EV	0.0980	0.1958	0.2724	0.3596	0.4643	0.5678
RMSE	0.4500	0.6455	0.7766	0.9028	1.0307	1.1519
RMSPE	0.0718	0.1080	0.1327	0.1564	0.1785	0.1971
MAE	0.3571	0.5093	0.6251	0.7400	0.8171	0.9103
MAPE	0.0578	0.0843	0.1046	0.1247	0.1385	0.1534

Table 6.3.13. Forecasting performance (Baa vs. Tsy), Engle-Granger, lag = 2

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0544	0.1101	0.1652	0.2075	0.2481	0.2898
EV	0.0425	0.0706	0.0839	0.0971	0.1217	0.1628
RMSE	0.2132	0.2876	0.3335	0.3744	0.4282	0.4967
RMSPE	0.0265	0.0360	0.0418	0.0470	0.0534	0.0616
MAE	0.1626	0.2236	0.2670	0.3053	0.3345	0.3740
MAPE	0.0205	0.0283	0.0338	0.0386	0.0420	0.0468
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0303)	(0.0765)	(0.1174)	(0.1533)	(0.1810)	(0.2031)
EV	0.0566	0.1269	0.1895	0.2566	0.3348	0.3965
RMSE	0.2398	0.3643	0.4508	0.5292	0.6063	0.6617
RMSPE	0.0456	0.0729	0.0926	0.1104	0.1261	0.1364
MAE	0.1975	0.2900	0.3655	0.4396	0.4934	0.5618
MAPE	0.0375	0.0566	0.0721	0.0872	0.0981	0.1107
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0241	0.0336	0.0478	0.0542	0.0671	0.0866
EV	0.0991	0.1975	0.2734	0.3537	0.4566	0.5593
RMSE	0.4530	0.6519	0.7843	0.9036	1.0345	1.1584
RMSPE	0.0721	0.1090	0.1344	0.1574	0.1796	0.1979
MAE	0.3600	0.5136	0.6325	0.7449	0.8279	0.9358
MAPE	0.0580	0.0849	0.1058	0.1258	0.1402	0.1575

Table 6.3.14. Forecasting performance (Baa vs. Tsy), Neal-Rolph-Morris, lag = 2

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0529	0.1036	0.1535	0.1916	0.2290	0.2678
EV	0.0426	0.0708	0.0836	0.0961	0.1195	0.1587
RMSE	0.2131	0.2855	0.3273	0.3645	0.4146	0.4800
RMSPE	0.0265	0.0358	0.0410	0.0457	0.0518	0.0595
MAE	0.1626	0.2227	0.2614	0.2968	0.3213	0.3571
MAPE	0.0205	0.0282	0.0331	0.0375	0.0404	0.0447
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0421)	(0.1039)	(0.1568)	(0.2015)	(0.2364)	(0.2648)
EV	0.0565	0.1264	0.1880	0.2541	0.3310	0.3908
RMSE	0.2415	0.3705	0.4611	0.5429	0.6220	0.6789
RMSPE	0.0461	0.0747	0.0957	0.1147	0.1312	0.1422
MAE	0.1993	0.2952	0.3699	0.4420	0.4995	0.5769
MAPE	0.0379	0.0579	0.0732	0.0884	0.1001	0.1146
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0108	(0.0003)	(0.0033)	(0.0099)	(0.0075)	0.0030
EV	0.0991	0.1972	0.2716	0.3503	0.4505	0.5495
RMSE	0.4546	0.6560	0.7884	0.9074	1.0366	1.1589
RMSPE	0.0726	0.1105	0.1367	0.1604	0.1830	0.2016
MAE	0.3618	0.5179	0.6314	0.7389	0.8208	0.9340
MAPE	0.0584	0.0861	0.1063	0.1259	0.1405	0.1593

Table 6.3.15. Forecasting performance (Aaa vs. Ibb), Lo-Zivot, lag = 1

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.1421	0.3090	0.4715	0.6877	0.9405	1.2354
EV	0.0251	0.0579	0.1022	0.1915	0.3661	0.6400
RMSE	0.2129	0.3916	0.5697	0.8151	1.1183	1.4719
RMSPE	0.0296	0.0548	0.0798	0.1145	0.1575	0.2077
MAE	0.1688	0.3276	0.4844	0.6978	0.9507	1.2524
MAPE	0.0238	0.0463	0.0683	0.0985	0.1342	0.1770
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.2248	0.4823	0.7323	1.0527	1.4195	1.8401
EV	0.0535	0.1618	0.2951	0.5697	1.0045	1.6192
RMSE	0.3225	0.6280	0.9118	1.2953	1.7377	2.2372
RMSPE	0.0555	0.1085	0.1574	0.2229	0.2985	0.3844
MAE	0.2600	0.5366	0.7795	1.1107	1.4930	1.9315
MAPE	0.0447	0.0925	0.1338	0.1903	0.2557	0.3310
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.3669	0.7913	1.2039	1.7404	2.3600	3.0755
EV	0.0786	0.2196	0.3972	0.7612	1.3706	2.2592
RMSE	0.5354	1.0196	1.4815	2.1104	2.8560	3.7091
RMSPE	0.0851	0.1634	0.2371	0.3374	0.4559	0.5921
MAE	0.4288	0.8642	1.2639	1.8085	2.4437	3.1838
MAPE	0.0686	0.1388	0.2022	0.2887	0.3899	0.5080

Table 6.3.16. Forecasting performance (Aaa vs. Ibb), Lo-Zivot, lag = 2

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0177	0.0909	0.1822	0.2819	0.4031	0.5458
EV	0.0320	0.0587	0.0788	0.1011	0.1616	0.2572
RMSE	0.1796	0.2587	0.3347	0.4250	0.5693	0.7450
RMSPE	0.0253	0.0367	0.0471	0.0597	0.0799	0.1045
MAE	0.1364	0.2041	0.2746	0.3569	0.4802	0.6185
MAPE	0.0194	0.0292	0.0391	0.0507	0.0680	0.0872
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0916	0.2287	0.3800	0.5480	0.7404	0.9597
EV	0.0444	0.1084	0.1748	0.2870	0.4669	0.7000
RMSE	0.2298	0.4008	0.5650	0.7664	1.0075	1.2732
RMSPE	0.0392	0.0692	0.0976	0.1315	0.1721	0.2173
MAE	0.1772	0.3258	0.4868	0.6524	0.8444	1.0762
MAPE	0.0303	0.0562	0.0836	0.1117	0.1442	0.1837
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.1093	0.3196	0.5622	0.8299	1.1435	1.5055
EV	0.0764	0.1670	0.2537	0.3881	0.6285	0.9571
RMSE	0.4094	0.6595	0.8997	1.1913	1.5768	2.0182
RMSPE	0.0645	0.1059	0.1446	0.1912	0.2520	0.3217
MAE	0.3135	0.5300	0.7614	1.0094	1.3246	1.6947
MAPE	0.0496	0.0854	0.1227	0.1624	0.2122	0.2709

Table 6.3.17. Forecasting performance (Aaa vs. Ibb), Hansen-Seo, lag = 1, β estimated

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0369	0.0516	0.0613	0.0635	0.0680	0.0754
EV	0.0302	0.0493	0.0647	0.0811	0.1131	0.1507
RMSE	0.1778	0.2279	0.2616	0.2918	0.3431	0.3954
RMSPE	0.0249	0.0324	0.0371	0.0414	0.0484	0.0553
MAE	0.1322	0.1828	0.2166	0.2262	0.2603	0.2952
MAPE	0.0187	0.0262	0.0310	0.0323	0.0371	0.0419
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0382)	(0.0789)	(0.1156)	(0.1518)	(0.1803)	(0.2013)
EV	0.0349	0.0716	0.1055	0.1474	0.1962	0.2400
RMSE	0.1907	0.2789	0.3448	0.4128	0.4782	0.5297
RMSPE	0.0330	0.0495	0.0617	0.0743	0.0858	0.0944
MAE	0.1602	0.2299	0.2802	0.3361	0.3935	0.4421
MAPE	0.0275	0.0401	0.0493	0.0591	0.0691	0.0775
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0013)	(0.0273)	(0.0543)	(0.0883)	(0.1123)	(0.1259)
EV	0.0651	0.1209	0.1702	0.2285	0.3093	0.3907
RMSE	0.3685	0.5068	0.6063	0.7047	0.8214	0.9251
RMSPE	0.0579	0.0819	0.0988	0.1157	0.1342	0.1497
MAE	0.2924	0.4126	0.4968	0.5624	0.6538	0.7373
MAPE	0.0463	0.0663	0.0803	0.0914	0.1062	0.1194

Table 6.3.18. Forecasting performance (Aaa vs. Ibb), Hansen-Seo, lag = 1, β = 1

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0179	(0.0110)	(0.0470)	(0.0859)	(0.1210)	(0.1523)
EV	0.0320	0.0494	0.0623	0.0788	0.1108	0.1483
RMSE	0.1797	0.2226	0.2539	0.2936	0.3541	0.4142
RMSPE	0.0253	0.0321	0.0368	0.0431	0.0521	0.0608
MAE	0.1371	0.1822	0.2095	0.2392	0.2925	0.3477
MAPE	0.0195	0.0263	0.0304	0.0350	0.0427	0.0508
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0706)	(0.1513)	(0.2217)	(0.2880)	(0.3463)	(0.3967)
EV	0.0351	0.0730	0.1085	0.1521	0.2023	0.2483
RMSE	0.2002	0.3097	0.3970	0.4848	0.5677	0.6369
RMSPE	0.0349	0.0554	0.0716	0.0879	0.1029	0.1151
MAE	0.1713	0.2514	0.3148	0.3975	0.4828	0.5429
MAPE	0.0296	0.0441	0.0557	0.0704	0.0854	0.0960
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0528)	(0.1623)	(0.2687)	(0.3739)	(0.4673)	(0.5490)
EV	0.0671	0.1225	0.1707	0.2309	0.3131	0.3966
RMSE	0.3799	0.5323	0.6509	0.7784	0.9218	1.0511
RMSPE	0.0602	0.0875	0.1085	0.1311	0.1550	0.1759
MAE	0.3084	0.4337	0.5244	0.6367	0.7753	0.8907
MAPE	0.0491	0.0704	0.0861	0.1053	0.1281	0.1467

Table 6.3.19. Forecasting performance (Aaa vs. Ibb), Enders-Siklos, M-C TAR, lag = 2

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0366	0.0469	0.0464	0.0375	0.0339	0.0344
EV	0.0324	0.0528	0.0667	0.0848	0.1165	0.1568
RMSE	0.1836	0.2345	0.2624	0.2936	0.3429	0.3974
RMSPE	0.0257	0.0334	0.0371	0.0417	0.0485	0.0559
MAE	0.1391	0.1863	0.2124	0.2283	0.2669	0.3045
MAPE	0.0197	0.0268	0.0304	0.0326	0.0381	0.0434
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0488)	(0.0994)	(0.1543)	(0.2036)	(0.2429)	(0.2740)
EV	0.0367	0.0735	0.1054	0.1486	0.1951	0.2438
RMSE	0.1976	0.2887	0.3595	0.4360	0.5040	0.5647
RMSPE	0.0343	0.0515	0.0646	0.0787	0.0908	0.1013
MAE	0.1632	0.2366	0.2948	0.3602	0.4305	0.4828
MAPE	0.0281	0.0413	0.0519	0.0634	0.0758	0.0849
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0122)	(0.0524)	(0.1079)	(0.1660)	(0.2090)	(0.2397)
EV	0.0690	0.1263	0.1721	0.2334	0.3115	0.4005
RMSE	0.3812	0.5232	0.6219	0.7295	0.8470	0.9621
RMSPE	0.0600	0.0848	0.1018	0.1204	0.1393	0.1572
MAE	0.3023	0.4229	0.5072	0.5885	0.6974	0.7873
MAPE	0.0478	0.0681	0.0823	0.0960	0.1139	0.1284

Table 6.3.20. Forecasting performance (Aaa vs. Ibb), Engle-Granger, lag = 2

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0366	0.0434	0.0404	0.0297	0.0246	0.0235
EV	0.0324	0.0519	0.0672	0.0859	0.1182	0.1573
RMSE	0.1838	0.2319	0.2623	0.2946	0.3447	0.3974
RMSPE	0.0257	0.0330	0.0371	0.0419	0.0489	0.0559
MAE	0.1392	0.1851	0.2121	0.2305	0.2660	0.3035
MAPE	0.0197	0.0266	0.0303	0.0330	0.0381	0.0433
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0554)	(0.1097)	(0.1682)	(0.2198)	(0.2611)	(0.2938)
EV	0.0363	0.0728	0.1083	0.1509	0.1976	0.2420
RMSE	0.1984	0.2912	0.3695	0.4463	0.5155	0.5730
RMSPE	0.0344	0.0520	0.0666	0.0808	0.0930	0.1029
MAE	0.1645	0.2388	0.2976	0.3673	0.4405	0.4910
MAPE	0.0284	0.0418	0.0525	0.0648	0.0776	0.0864
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0188)	(0.0664)	(0.1277)	(0.1900)	(0.2365)	(0.2703)
EV	0.0687	0.1247	0.1754	0.2368	0.3158	0.3993
RMSE	0.3822	0.5232	0.6318	0.7409	0.8602	0.9703
RMSPE	0.0602	0.0850	0.1037	0.1227	0.1419	0.1588
MAE	0.3037	0.4239	0.5096	0.5978	0.7065	0.7946
MAPE	0.0481	0.0684	0.0828	0.0977	0.1157	0.1298

Table 6.3.21. Forecasting performance (Baa vs. Ibb), Lo-Zivot, lag = 1

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	(0.0130)	(0.0269)	(0.0412)	(0.0349)	(0.0245)	(0.0177)
EV	0.0277	0.0453	0.0593	0.0684	0.0911	0.1189
RMSE	0.1670	0.2145	0.2469	0.2638	0.3028	0.3453
RMSPE	0.0209	0.0275	0.0319	0.0345	0.0394	0.0445
MAE	0.1221	0.1741	0.2029	0.2126	0.2597	0.2875
MAPE	0.0155	0.0223	0.0260	0.0275	0.0335	0.0371
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0307	0.0494	0.0677	0.0602	0.0545	0.0570
EV	0.0404	0.0876	0.1413	0.1927	0.2383	0.2697
RMSE	0.2034	0.3000	0.3819	0.4431	0.4912	0.5224
RMSPE	0.0347	0.0516	0.0656	0.0766	0.0848	0.0892
MAE	0.1658	0.2528	0.3184	0.3647	0.3927	0.4207
MAPE	0.0283	0.0434	0.0546	0.0628	0.0675	0.0720
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0178	0.0225	0.0264	0.0253	0.0299	0.0393
EV	0.0681	0.1329	0.2006	0.2610	0.3294	0.3885
RMSE	0.3704	0.5145	0.6288	0.7068	0.7941	0.8677
RMSPE	0.0556	0.0791	0.0975	0.1111	0.1241	0.1338
MAE	0.2879	0.4268	0.5213	0.5773	0.6524	0.7083
MAPE	0.0438	0.0657	0.0807	0.0903	0.1010	0.1090

Table 6.3.22. Forecasting performance (Baa vs. Ibb), Lo-Zivot, lag = 2

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0106	0.0283	0.0489	0.0486	0.0531	0.0642
EV	0.0367	0.0635	0.0724	0.0853	0.1028	0.1328
RMSE	0.1918	0.2535	0.2735	0.2961	0.3249	0.3700
RMSPE	0.0236	0.0316	0.0342	0.0373	0.0409	0.0467
MAE	0.1388	0.2024	0.2255	0.2383	0.2556	0.2851
MAPE	0.0174	0.0256	0.0285	0.0302	0.0324	0.0362
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0031	0.0000	(0.0031)	(0.0156)	(0.0286)	(0.0314)
EV	0.0458	0.1048	0.1577	0.2067	0.2661	0.3089
RMSE	0.2140	0.3238	0.3971	0.4549	0.5166	0.5566
RMSPE	0.0364	0.0563	0.0692	0.0797	0.0907	0.0980
MAE	0.1740	0.2717	0.3222	0.3690	0.4095	0.4300
MAPE	0.0297	0.0470	0.0560	0.0642	0.0712	0.0745
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0137	0.0283	0.0458	0.0330	0.0245	0.0328
EV	0.0825	0.1683	0.2301	0.2920	0.3689	0.4416
RMSE	0.4058	0.5773	0.6706	0.7509	0.8416	0.9266
RMSPE	0.0601	0.0879	0.1033	0.1170	0.1316	0.1447
MAE	0.3128	0.4741	0.5477	0.6073	0.6651	0.7151
MAPE	0.0472	0.0726	0.0845	0.0944	0.1036	0.1107

Table 6.3.23. Forecasting performance (Baa vs. Ibb), Hansen-Seo, lag = 1, β estimated

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0507	0.0974	0.1296	0.1436	0.1536	0.1619
EV	0.0330	0.0534	0.0700	0.0824	0.1067	0.1387
RMSE	0.1887	0.2508	0.2946	0.3209	0.3609	0.4061
RMSPE	0.0234	0.0316	0.0370	0.0405	0.0454	0.0506
MAE	0.1391	0.1934	0.2389	0.2625	0.2877	0.3041
MAPE	0.0175	0.0245	0.0303	0.0334	0.0363	0.0382
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0253)	(0.0608)	(0.0992)	(0.1364)	(0.1672)	(0.1943)
EV	0.0433	0.0746	0.1100	0.1525	0.1998	0.2434
RMSE	0.2097	0.2799	0.3461	0.4137	0.4772	0.5302
RMSPE	0.0370	0.0500	0.0622	0.0744	0.0854	0.0941
MAE	0.1687	0.2265	0.2926	0.3460	0.4021	0.4357
MAPE	0.0293	0.0397	0.0516	0.0610	0.0706	0.0763
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0253	0.0365	0.0304	0.0072	(0.0136)	(0.0323)
EV	0.0763	0.1281	0.1799	0.2349	0.3065	0.3821
RMSE	0.3983	0.5307	0.6407	0.7346	0.8381	0.9363
RMSPE	0.0604	0.0815	0.0992	0.1149	0.1308	0.1447
MAE	0.3078	0.4199	0.5316	0.6085	0.6898	0.7398
MAPE	0.0468	0.0642	0.0819	0.0944	0.1070	0.1146

Table 6.3.24. Forecasting performance (Baa vs. Ibb), Hansen-Seo, lag = 1, β = 1

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0248	0.0521	0.0768	0.0916	0.1075	0.1295
EV	0.0379	0.0585	0.0618	0.0615	0.0724	0.0980
RMSE	0.1961	0.2475	0.2601	0.2643	0.2898	0.3388
RMSPE	0.0242	0.0309	0.0325	0.0335	0.0366	0.0422
MAE	0.1450	0.1952	0.2132	0.2166	0.2392	0.2604
MAPE	0.0182	0.0247	0.0270	0.0276	0.0304	0.0328
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0072)	(0.0210)	(0.0369)	(0.0511)	(0.0591)	(0.0622)
EV	0.0356	0.0704	0.1003	0.1371	0.1803	0.2229
RMSE	0.1889	0.2662	0.3189	0.3738	0.4287	0.4762
RMSPE	0.0327	0.0471	0.0569	0.0669	0.0762	0.0835
MAE	0.1531	0.2131	0.2690	0.3169	0.3533	0.3834
MAPE	0.0264	0.0372	0.0472	0.0556	0.0618	0.0667
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0176	0.0311	0.0399	0.0405	0.0484	0.0673
EV	0.0735	0.1289	0.1621	0.1985	0.2527	0.3209
RMSE	0.3851	0.5137	0.5790	0.6380	0.7185	0.8150
RMSPE	0.0570	0.0781	0.0894	0.1003	0.1128	0.1257
MAE	0.2981	0.4083	0.4822	0.5334	0.5925	0.6439
MAPE	0.0446	0.0619	0.0742	0.0831	0.0921	0.0995

Table 6.3.25. Forecasting performance (Baa vs. Ibb), Enders-Siklos, C-TAR, lag = 1

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0245	0.0377	0.0495	0.0502	0.0542	0.0667
EV	0.0400	0.0658	0.0743	0.0860	0.1063	0.1369
RMSE	0.2015	0.2593	0.2770	0.2975	0.3305	0.3759
RMSPE	0.0249	0.0325	0.0346	0.0375	0.0416	0.0470
MAE	0.1525	0.2018	0.2098	0.2423	0.2571	0.2838
MAPE	0.0192	0.0255	0.0265	0.0308	0.0326	0.0359
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0489)	(0.1015)	(0.1496)	(0.1953)	(0.2322)	(0.2602)
EV	0.0348	0.0711	0.1053	0.1485	0.1975	0.2404
RMSE	0.1927	0.2853	0.3574	0.4320	0.5014	0.5551
RMSPE	0.0335	0.0511	0.0648	0.0790	0.0915	0.1009
MAE	0.1646	0.2325	0.2810	0.3461	0.4121	0.4587
MAPE	0.0284	0.0407	0.0498	0.0614	0.0730	0.0812
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0244)	(0.0637)	(0.1001)	(0.1451)	(0.1779)	(0.1936)
EV	0.0748	0.1369	0.1796	0.2345	0.3038	0.3773
RMSE	0.3943	0.5446	0.6344	0.7295	0.8319	0.9310
RMSPE	0.0585	0.0836	0.0994	0.1165	0.1331	0.1479
MAE	0.3171	0.4343	0.4908	0.5884	0.6692	0.7425
MAPE	0.0476	0.0663	0.0763	0.0921	0.1056	0.1170

Table 6.3.26. Forecasting performance (Baa vs. Ibb), Engle-Granger, lag = 2

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0298	0.0472	0.0603	0.0604	0.0626	0.0717
EV	0.0406	0.0658	0.0723	0.0805	0.0986	0.1274
RMSE	0.2037	0.2608	0.2756	0.2901	0.3201	0.3640
RMSPE	0.0252	0.0327	0.0344	0.0366	0.0403	0.0455
MAE	0.1541	0.2037	0.2152	0.2377	0.2487	0.2762
MAPE	0.0194	0.0258	0.0272	0.0302	0.0315	0.0349
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0480)	(0.1003)	(0.1484)	(0.1940)	(0.2310)	(0.2593)
EV	0.0347	0.0709	0.1049	0.1477	0.1963	0.2389
RMSE	0.1924	0.2846	0.3562	0.4305	0.4997	0.5533
RMSPE	0.0335	0.0510	0.0646	0.0787	0.0912	0.1006
MAE	0.1640	0.2320	0.2804	0.3450	0.4112	0.4580
MAPE	0.0283	0.0406	0.0496	0.0611	0.0728	0.0810
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0182)	(0.0531)	(0.0881)	(0.1337)	(0.1684)	(0.1876)
EV	0.0753	0.1367	0.1772	0.2282	0.2949	0.3662
RMSE	0.3961	0.5453	0.6318	0.7207	0.8198	0.9173
RMSPE	0.0586	0.0836	0.0990	0.1153	0.1315	0.1461
MAE	0.3181	0.4358	0.4956	0.5827	0.6599	0.7342
MAPE	0.0477	0.0664	0.0769	0.0913	0.1044	0.1159

Table 6.4.1. Results of Lo-Zivot models with non-unity cointegrating vectors

	Aaa vs. Tsy		Aaa vs. Tsy		Baa vs. Tsy	
Cointegrating vector	[1, -1.028]		[1, -1.039]		[1, -1.178]	
1st regime short rate eq	**est. coeff.**	**std error**	**est. coeff.**	**std error**	**est. coeff.**	**std error**
constant term	0.0564	0.0402	0.0741	0.0484	0.0967	0.0634
error-correction term	0.4051	0.2750	0.2234	0.2132	0.3043	0.1516
short rate lagged one term	1.2385	0.2603	1.3281	0.2308	1.1612	0.1740
long rate lagged one term	(1.2162)	0.3027	(1.4071)	0.2690	(1.0680)	0.2260
1st regime long rate eq						
constant term	0.0013	0.0922	(0.0072)	0.0871	0.0003	0.0269
error-correction term	(0.1265)	0.4440	0.0279	0.7990	0.0544	0.0758
short rate lagged one term	0.9758	0.2690	0.7780	0.3310	0.2361	0.1064
long rate lagged one term	(0.3533)	0.3209	0.1015	0.3907	0.0393	0.1571
2nd regime short rate eq						
constant term	(0.0545)	0.0455	(0.0361)	0.0358	(0.2383)	0.0969
error-correction term	0.0803	0.0611	0.0671	0.0554	0.2219	0.0840
short rate lagged one term	0.3931	0.1214	0.3853	0.1179	0.6055	0.1221
long rate lagged one term	(0.1638)	0.1615	(0.1423)	0.1585	(0.2089)	0.1842
2nd regime long rate eq						
constant term	0.0708	0.0295	0.0849	0.0355	0.0556	0.0407
error-correction term	0.0876	0.2019	0.0708	0.1564	0.0844	0.0972
short rate lagged one term	1.1831	0.1911	1.2602	0.1693	0.7309	0.1116
long rate lagged one term	(1.1040)	0.2223	(1.2276)	0.1973	(0.3019)	0.1450
3rd regime short rate eq						
constant term	0.0050	0.0677	(0.0245)	0.0639	0.0103	0.0173
error-correction term	(0.0678)	0.3260	0.2805	0.5860	(0.0175)	0.0487
short rate lagged one term	0.8638	0.1975	0.5819	0.2427	0.1616	0.0683
long rate lagged one term	(0.2722)	0.2356	0.2079	0.2866	0.2091	0.1008
3rd regime long rate eq						
constant term	(0.0259)	0.0334	(0.0131)	0.0263	(0.0393)	0.0622
error-correction term	0.0212	0.0449	0.0091	0.0406	0.0200	0.0539
short rate lagged one term	0.3084	0.0892	0.3210	0.0865	0.2945	0.0784
long rate lagged one term	(0.0477)	0.1186	(0.0556)	0.1162	0.2089	0.1181
1st regime obs	68		70		75	
2nd regime obs	96		71		271	
3rd regime obs	290		313		108	
Gamma hat 1	0.09		0.03		(0.15)	
Gamma hat 2	0.31		0.18		0.67	
LR13	52.09		67.34		56.32	
p-value	0.004		0.001		0.002	

Table 6.4.1. (continued)

	Baa vs. Tsy		Aaa vs. Ibb		Baa vs. Ibb	
Cointegrating vector	[1, -1.108]		[1, -0.981]		[1, -1.385]	
1st regime short rate eq	**est. coeff.**	**std error**	**est. coeff.**	**std error**	**est. coeff.**	**std error**
constant term	0.0166	0.0576	(0.0261)	0.0598	0.3481	0.1180
error-correction term	0.0057	0.2382	0.0266	0.1113	0.1712	0.0535
short rate lagged one term	1.3569	0.2118	0.1659	0.0983	0.4185	0.0821
long rate lagged one term	(1.5600)	0.2556	0.1811	0.1210	(0.2055)	0.1103
1st regime long rate eq						
constant term	0.0486	0.0499	(0.5934)	0.6923	0.1262	0.0773
error-correction term	(0.0573)	0.0550	0.6010	0.8317	0.1140	0.0672
short rate lagged one term	0.4597	0.0951	(0.4351)	0.1633	0.1174	0.0772
long rate lagged one term	(0.0866)	0.1369	(0.0828)	0.1631	0.0123	0.1130
2nd regime short rate eq						
constant term	0.1843	0.2110	(0.0912)	0.1724	(0.0267)	0.0513
error-correction term	(0.0726)	0.1151	0.1180	0.1408	(0.1058)	0.1487
short rate lagged one term	0.2058	0.1367	0.2677	0.0953	(0.5446)	0.1652
long rate lagged one term	0.0796	0.2152	(0.2284)	0.1137	0.0427	0.2968
2nd regime long rate eq						
constant term	0.0579	0.0364	0.0495	0.0327	0.0339	0.0552
error-correction term	(0.2036)	0.1507	(0.0830)	0.0608	0.0111	0.0250
short rate lagged one term	0.8000	0.1340	0.5108	0.0537	0.5220	0.0384
long rate lagged one term	(0.4499)	0.1617	0.1038	0.0661	0.1066	0.0516
3rd regime short rate eq						
constant term	0.0504	0.0316	(0.7287)	0.3781	0.0605	0.0362
error-correction term	(0.0574)	0.0348	0.8251	0.4542	0.0602	0.0315
short rate lagged one term	0.3309	0.0602	0.1685	0.0892	0.4059	0.0361
long rate lagged one term	0.1307	0.0866	0.0615	0.0891	0.2789	0.0529
3rd regime long rate eq						
constant term	0.1884	0.1335	0.0897	0.0942	(0.0741)	0.0240
error-correction term	(0.1198)	0.0728	(0.0887)	0.0769	(0.1802)	0.0696
short rate lagged one term	(0.0089)	0.0865	0.5622	0.0521	(0.0369)	0.0773
long rate lagged one term	0.3851	0.1362	(0.1669)	0.0621	0.2137	0.1389
1st regime obs	78		284		158	
2nd regime obs	309		73		225	
3rd regime obs	67		97		71	
Gamma hat 1	0.35		0.76		(1.62)	
Gamma hat 2	1.48		0.92		(0.52)	
LR13	73.03		47.50		70.75	
p-value	0		0.011		0	

Table 6.4.2. Forecasting performance (Aaa vs. Tsy), Lo-Zivot, lag = 2, β= 1.028

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	(0.0055)	0.0170	0.0625	0.1094	0.1723	0.2516
EV	0.0297	0.0585	0.0816	0.0990	0.1414	0.1967
RMSE	0.1724	0.2424	0.2924	0.3331	0.4136	0.5099
RMSPE	0.0245	0.0347	0.0417	0.0473	0.0581	0.0709
MAE	0.1380	0.1929	0.2426	0.2622	0.3125	0.3671
MAPE	0.0198	0.0278	0.0348	0.0375	0.0442	0.0516
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0890	0.2612	0.4875	0.7607	1.0977	1.5105
EV	0.0749	0.2239	0.3932	0.6227	1.0047	1.5333
RMSE	0.2878	0.5405	0.7943	1.0960	1.4865	1.9532
RMSPE	0.0547	0.1054	0.1565	0.2127	0.2827	0.3690
MAE	0.2296	0.4307	0.6291	0.8775	1.1942	1.5847
MAPE	0.0435	0.0827	0.1207	0.1673	0.2254	0.2993
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0834	0.2782	0.5500	0.8701	1.2700	1.7622
EV	0.1046	0.2824	0.4748	0.7217	1.1461	1.7300
RMSE	0.4601	0.7829	1.0867	1.4292	1.9001	2.4631
RMSPE	0.0792	0.1402	0.1981	0.2600	0.3408	0.4399
MAE	0.3676	0.6236	0.8717	1.1397	1.5066	1.9519
MAPE	0.0633	0.1105	0.1555	0.2048	0.2696	0.3508

Table 6.4.3. Forecasting performance (Aaa vs. Tsy), Lo-Zivot, lag = 2, β = 1.039

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0003	(0.1127)	(0.2493)	(0.5518)	(0.8429)	(0.9653)
EV	0.0300	0.0608	0.0928	0.2092	0.2659	0.4091
RMSE	0.1731	0.2710	0.3937	0.7167	0.9881	1.1580
RMSPE	0.0246	0.0393	0.0575	0.1052	0.1446	0.1697
MAE	0.1373	0.2162	0.3166	0.5879	0.8561	0.9863
MAPE	0.0196	0.0314	0.0461	0.0858	0.1247	0.1439
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.3820)	(0.8597)	(1.4086)	(1.8004)	(2.0598)	(2.1617)
EV	0.0680	0.1789	0.3464	0.5087	0.7466	0.7985
RMSE	0.4625	0.9581	1.5266	1.9366	2.2337	2.3391
RMSPE	0.0917	0.1935	0.3093	0.3928	0.4564	0.4798
MAE	0.3975	0.8634	1.4086	1.8004	2.0598	2.1617
MAPE	0.0776	0.1698	0.2773	0.3541	0.4068	0.4281
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.3816)	(0.9724)	(1.6579)	(2.3522)	(2.9028)	(3.1271)
EV	0.0980	0.2397	0.4392	0.7179	1.0125	1.2075
RMSE	0.6357	1.2291	1.9203	2.6533	3.2218	3.4971
RMSPE	0.1163	0.2328	0.3668	0.4980	0.6011	0.6495
MAE	0.5348	1.0795	1.7252	2.3883	2.9159	3.1480
MAPE	0.0973	0.2012	0.3233	0.4399	0.5315	0.5721

Table 6.4.4. Forecasting performance (Baa vs. Tsy), Lo-Zivot, lag = 2, β = 1.178

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0022	0.0189	0.0405	0.0541	0.0701	0.0919
EV	0.0453	0.0771	0.0900	0.0991	0.1275	0.1655
RMSE	0.2127	0.2783	0.3027	0.3195	0.3639	0.4171
RMSPE	0.0265	0.0350	0.0381	0.0406	0.0459	0.0521
MAE	0.1641	0.2222	0.2447	0.2646	0.2882	0.3182
MAPE	0.0208	0.0282	0.0311	0.0337	0.0365	0.0402
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0030	0.0002	(0.0019)	(0.0124)	(0.0176)	(0.0136)
EV	0.0532	0.1244	0.1907	0.2576	0.3452	0.4064
RMSE	0.2306	0.3527	0.4367	0.5077	0.5878	0.6376
RMSPE	0.0437	0.0693	0.0873	0.1027	0.1182	0.1267
MAE	0.1885	0.2797	0.3575	0.4249	0.4775	0.5213
MAPE	0.0358	0.0542	0.0699	0.0835	0.0937	0.1013
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0052	0.0190	0.0385	0.0417	0.0525	0.0783
EV	0.0984	0.2015	0.2807	0.3568	0.4727	0.5719
RMSE	0.4433	0.6310	0.7394	0.8272	0.9517	1.0547
RMSPE	0.0702	0.1043	0.1255	0.1433	0.1641	0.1789
MAE	0.3526	0.5019	0.6022	0.6896	0.7657	0.8396
MAPE	0.0566	0.0824	0.1010	0.1173	0.1302	0.1414

Table 6.4.5. Forecasting performance (Baa vs. Tsy), Lo-Zivot, lag = 2, β = 1.108

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0959	0.1638	0.2578	0.3158	0.3641	0.4114
EV	0.0406	0.0703	0.0946	0.1243	0.1779	0.2752
RMSE	0.2231	0.3117	0.4013	0.4733	0.5572	0.6667
RMSPE	0.0279	0.0389	0.0507	0.0598	0.0701	0.0835
MAE	0.1741	0.2291	0.3228	0.3845	0.4473	0.5281
MAPE	0.0221	0.0289	0.0409	0.0487	0.0564	0.0665
Tsy	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.0115	0.0294	0.0445	0.0365	0.0253	0.0205
EV	0.0663	0.1828	0.3249	0.5303	0.8192	1.1136
RMSE	0.2577	0.4285	0.5718	0.7292	0.9055	1.0555
RMSPE	0.0486	0.0837	0.1122	0.1412	0.1733	0.2008
MAE	0.2091	0.3476	0.4949	0.6299	0.7749	0.8864
MAPE	0.0394	0.0671	0.0954	0.1207	0.1470	0.1674
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	0.1074	0.1932	0.3023	0.3523	0.3894	0.4319
EV	0.1068	0.2531	0.4195	0.6546	0.9971	1.3888
RMSE	0.4807	0.7403	0.9731	1.2024	1.4627	1.7221
RMSPE	0.0765	0.1225	0.1628	0.2010	0.2434	0.2843
MAE	0.3833	0.5767	0.8177	1.0144	1.2222	1.4145
MAPE	0.0615	0.0960	0.1363	0.1694	0.2034	0.2339

Table 6.4.6. Forecasting performance (Aaa vs. Ibb), Lo-Zivot, lag = 2, β = 0.981

Aaa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0262	(0.1091)	(0.1335)	(0.1209)	(0.0985)	(0.0213)
EV	0.0316	0.0921	0.1362	0.1919	0.3201	0.5173
RMSE	0.1797	0.3225	0.3925	0.4544	0.5743	0.7196
RMSPE	0.0254	0.0464	0.0564	0.0657	0.0834	0.1055
MAE	0.1358	0.2525	0.3129	0.3694	0.4862	0.6029
MAPE	0.0193	0.0364	0.0450	0.0534	0.0705	0.0878
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.3028)	(0.4131)	(0.4741)	(0.5519)	(0.5839)	(0.5964)
EV	0.0932	0.1705	0.2204	0.3214	0.4062	0.4916
RMSE	0.4300	0.5840	0.6672	0.7912	0.8644	0.9205
RMSPE	0.0755	0.1031	0.1182	0.1404	0.1537	0.1636
MAE	0.3516	0.4813	0.5472	0.6446	0.7045	0.7551
MAPE	0.0614	0.0844	0.0963	0.1135	0.1240	0.1327
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.2766)	(0.5222)	(0.6076)	(0.6728)	(0.6824)	(0.6178)
EV	0.1248	0.2626	0.3566	0.5133	0.7263	1.0089
RMSE	0.6097	0.9066	1.0597	1.2456	1.4387	1.6401
RMSPE	0.1009	0.1494	0.1746	0.2062	0.2371	0.2691
MAE	0.4874	0.7338	0.8601	1.0139	1.1907	1.3580
MAPE	0.0808	0.1208	0.1413	0.1669	0.1945	0.2205

Table 6.4.7. Forecasting performance (Baa vs. Ibb), Lo-Zivot, lag = 2, β = 1.385

Baa	1-step	2-step	3-step	4-step	5-step	6-step
ME	0.0425	0.0659	0.0946	0.0769	0.0502	0.0169
EV	0.0335	0.0635	0.0823	0.0964	0.1194	0.1546
RMSE	0.1878	0.2604	0.3020	0.3199	0.3491	0.3936
RMSPE	0.0233	0.0327	0.0382	0.0407	0.0444	0.0501
MAE	0.1384	0.1898	0.2461	0.2592	0.2871	0.3241
MAPE	0.0175	0.0240	0.0313	0.0331	0.0366	0.0413
Ibb	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0624)	(0.1149)	(0.1777)	(0.2388)	(0.3034)	(0.3524)
EV	0.0455	0.0889	0.1280	0.1746	0.2355	0.2856
RMSE	0.2222	0.3195	0.3995	0.4813	0.5723	0.6402
RMSPE	0.0381	0.0568	0.0717	0.0872	0.1038	0.1164
MAE	0.1804	0.2660	0.3191	0.4021	0.4852	0.5464
MAPE	0.0310	0.0466	0.0562	0.0709	0.0857	0.0966
Sum	**1-step**	**2-step**	**3-step**	**4-step**	**5-step**	**6-step**
ME	(0.0199)	(0.0490)	(0.0832)	(0.1619)	(0.2532)	(0.3355)
EV	0.0790	0.1524	0.2103	0.2711	0.3549	0.4402
RMSE	0.4100	0.5800	0.7016	0.8013	0.9214	1.0337
RMSPE	0.0614	0.0895	0.1099	0.1279	0.1482	0.1665
MAE	0.3189	0.4558	0.5652	0.6614	0.7723	0.8705
MAPE	0.0485	0.0706	0.0875	0.1040	0.1223	0.1379

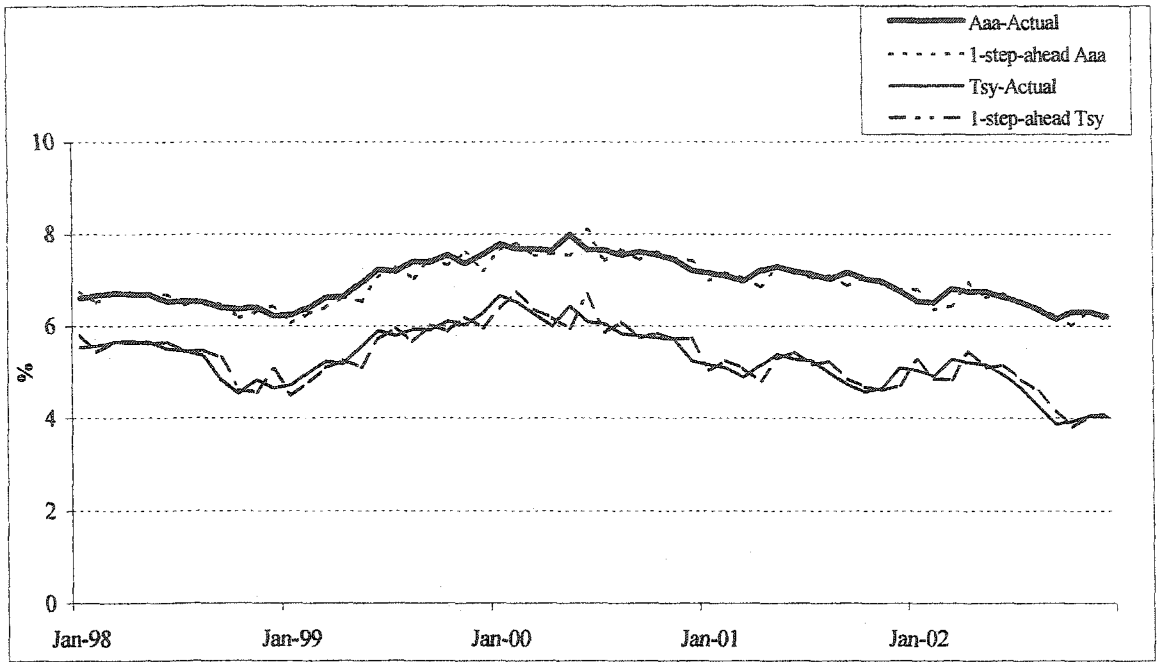

Figure 6.1. Aaa vs. Tsy, Enders-Siklos, M-C TAR, lag = 2

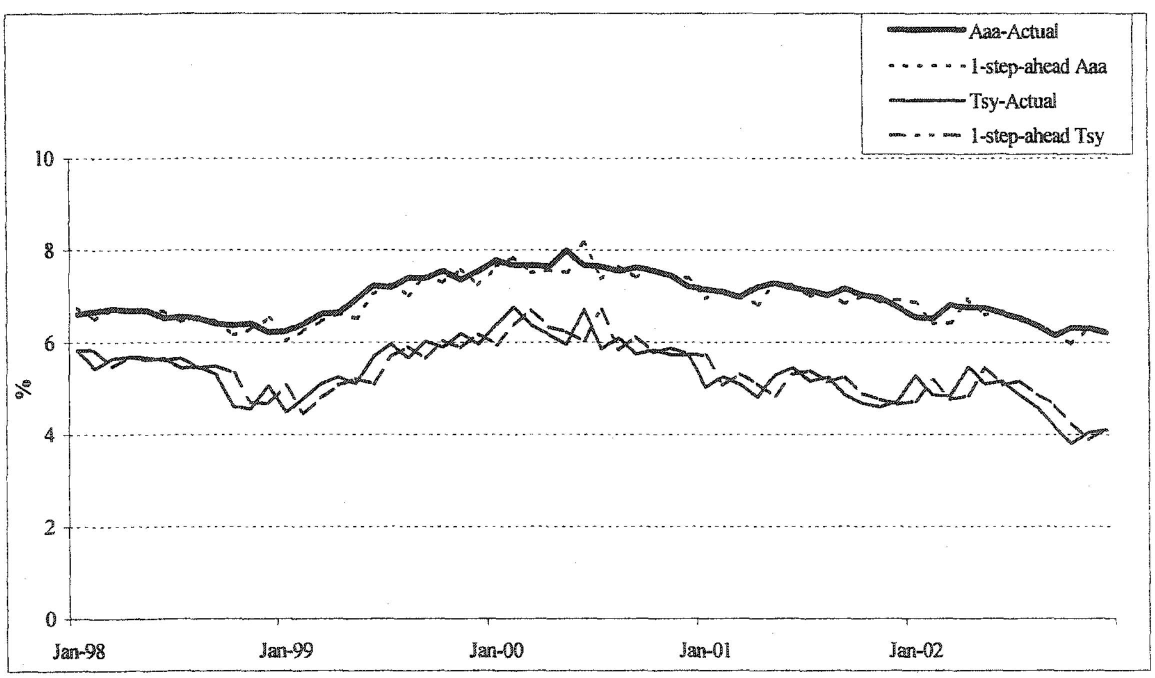

Figure 6.2. Aaa vs. Tsy, Engle-Granger, lag = 2

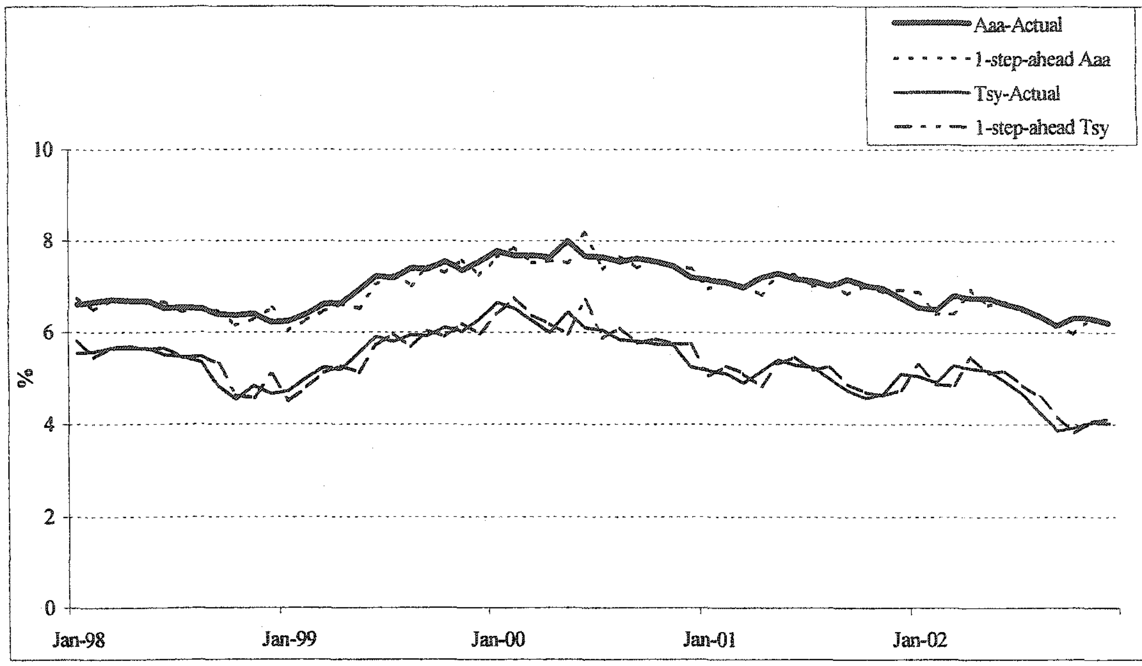

Figure 6.3. Aaa vs. Tsy, Neal-Rolph-Morris, lag = 2

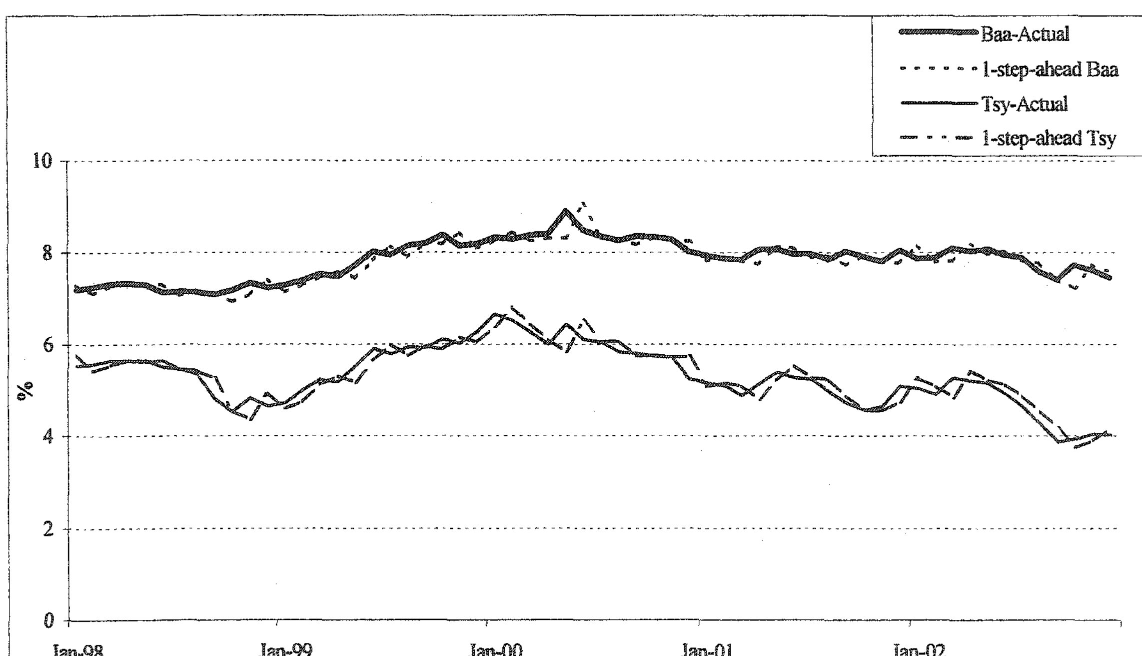

Figure 6.4. Baa vs. Tsy, Hansen-Seo, lag = 1, β = 1

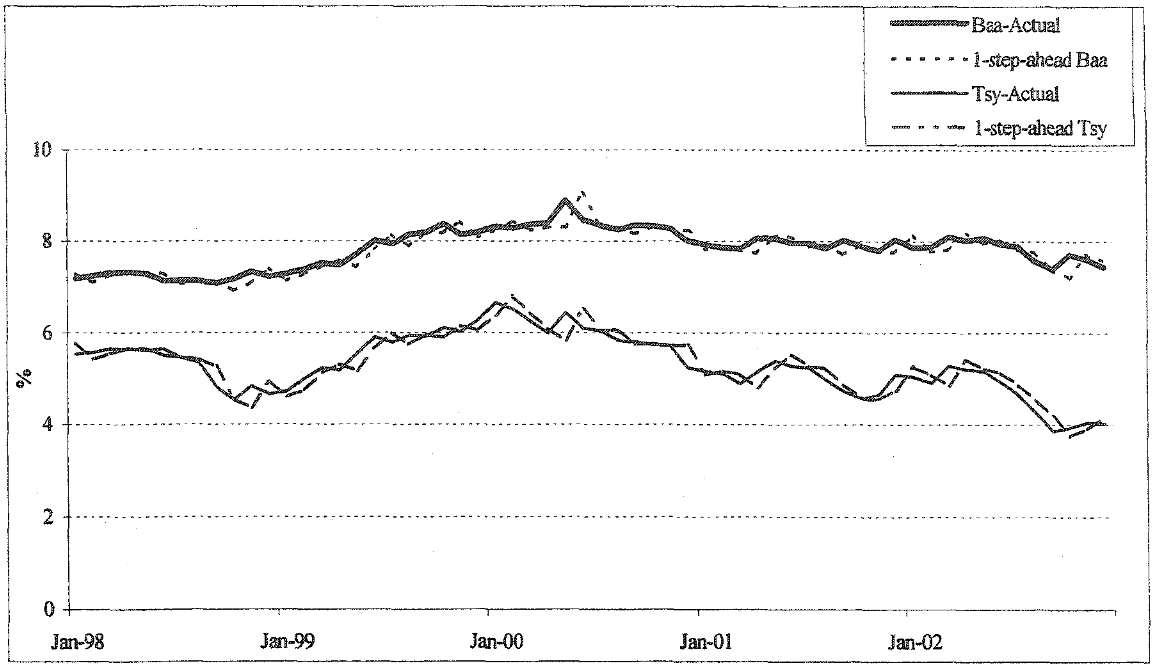

Figure 6.5. Baa vs. Tsy, Hansen-Seo, lag = 1, β estimated

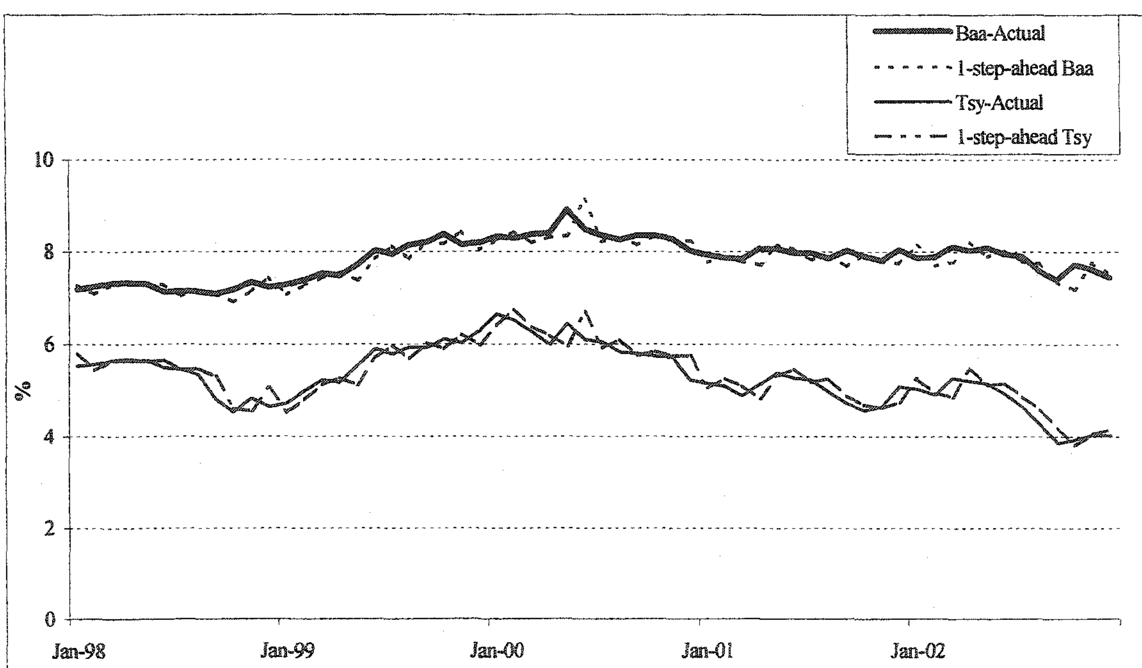

Figure 6.6. Baa vs. Tsy, Neal-Rolph-Morris, lag = 2

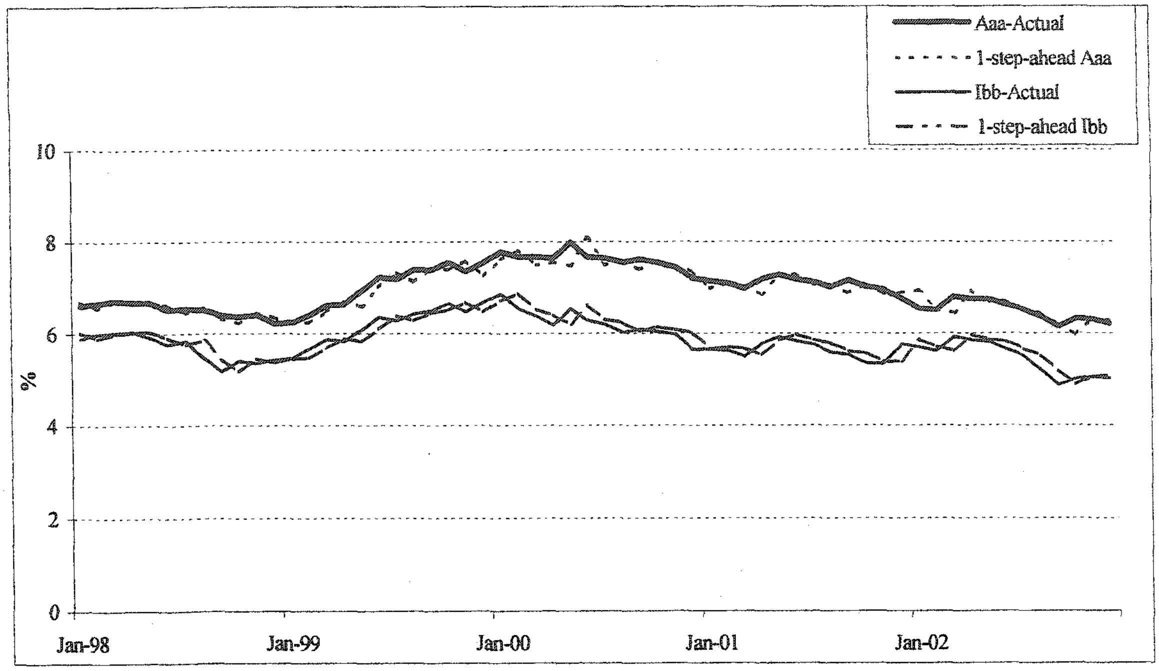

Figure 6.7. Aaa vs. Ibb, Hansen-Seo, lag = 1, β estimated

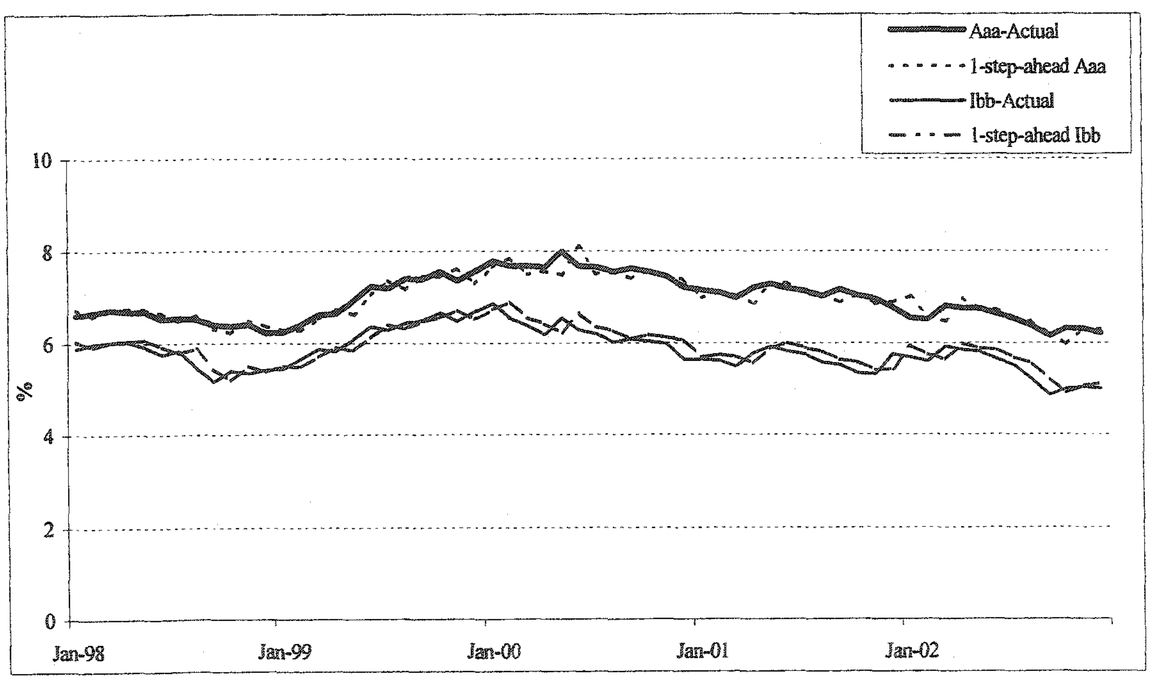

Figure 6.8. Aaa vs. Ibb, Hansen-Seo, lag = 1, β = 1

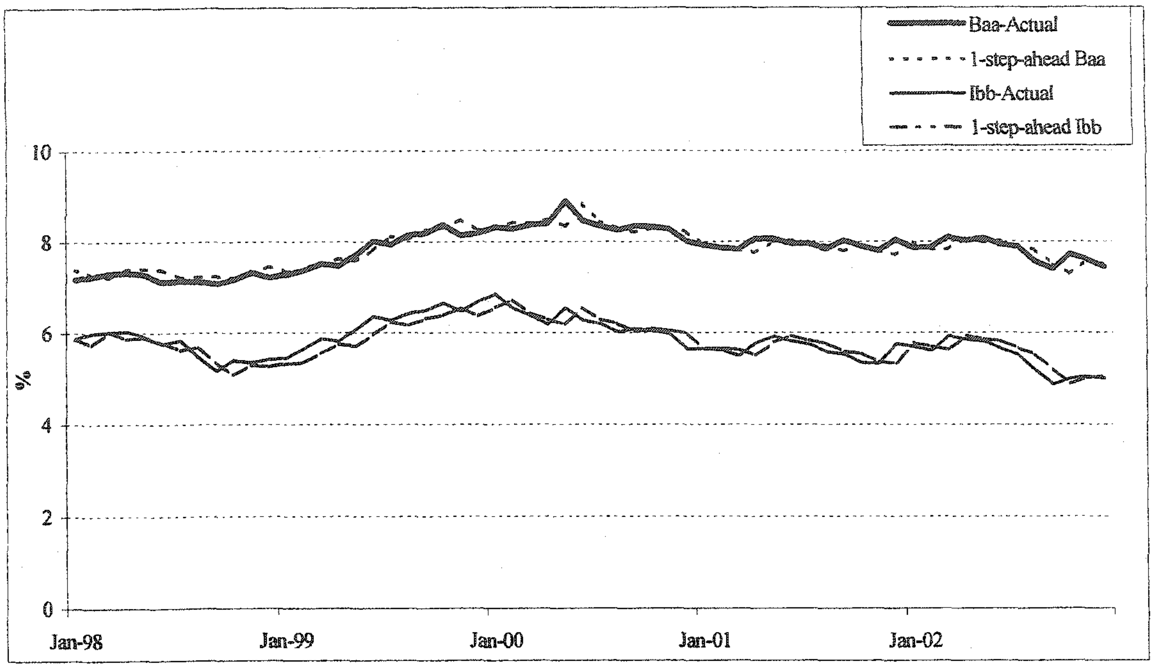

Figure 6.9. Baa vs. Ibb, Lo-Zivot, lag = 1

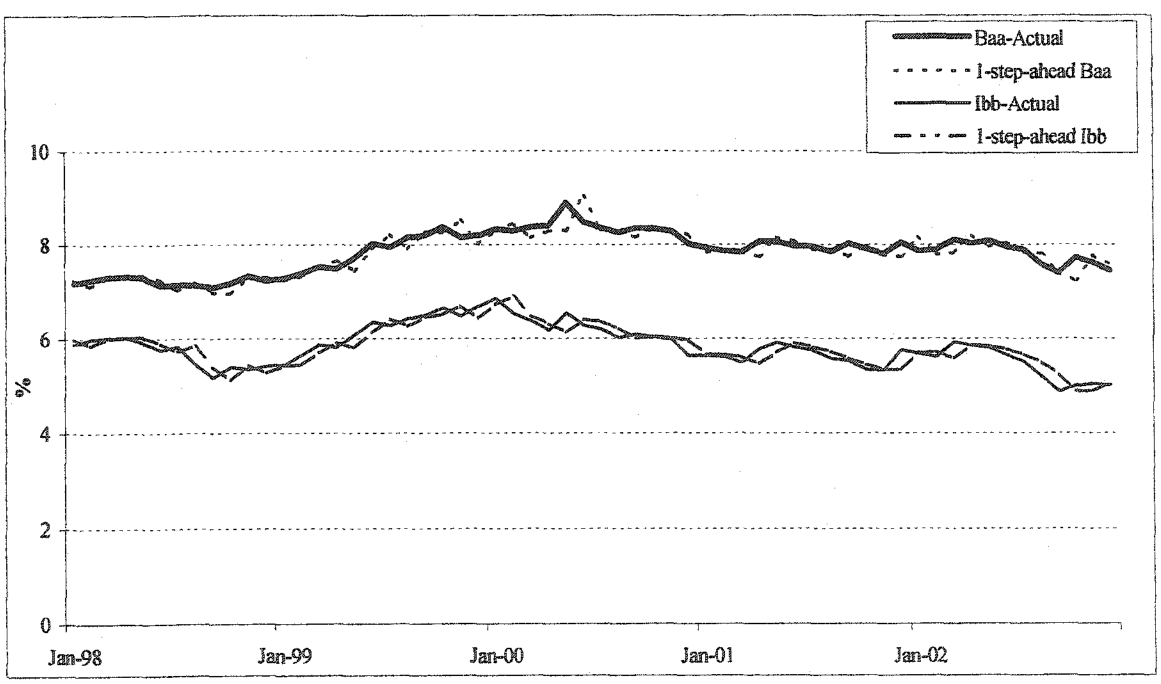

Figure 6.10. Baa vs. Ibb, Hansen-Seo, lag = 1, $\beta = 1$

CHAPTER 7. CONCLUSIONS

This study utilized monthly averages of daily rates for the 10-year constant maturity Treasury note, the Ibbotson Bond Index with maturity of 20-year Treasury Index, and Moody's Aaa and Baa seasoned bond indices to investigate the threshold behavior of interest rates pairs. The data covered the period from January 1960 to December 1997 with a total of 456 observations for each variable. Three different non-linear, discontinuous, asymmetric time-series econometric alternatives were applied to investigate the dynamics of the four interest rates pairs. Forecasting accuracy evaluation was utilized for model evaluation by applying one-step-ahead up to six-step-ahead forecasts.

Among the findings, it was ascertained that interest spreads are stationary, yet the speeds of adjustment are asymmetric. In a bivariate setting, all of the interest rates pairs followed the threshold cointegration behavior. All the interest rates pairs were shown to be threshold cointegrated. In general, the adjustment speeds were asymmetric and, especially, the threshold estimates were asymmetric in a three-regime environment.

Long run equilibrium relationships existed between Moody's corporate bond indices and Treasury note and Ibbotson bond index. In general, for a one percent increase in Treasury rates (either Treasury note or Ibbotson index), in the long run, it will generate a more than one percent increase in corporate bond indices (Aaa or Baa). Furthermore, the Baa bond index was shown to have a greater sensitivity to interest rate changes than the Aaa bond index. Above findings are coherent with NRM(2000) but inconsistent with a commonly held view that increased credit risk will make corporate bonds less interest rate sensitive as stressed by NRM(2000). This finding has strong implications in the area of corporate bond pricing, risk management, and portfolio management.

For the model evaluation side, one-step-ahead forecast to six-step-ahead forecast performance evaluations were conducted for the threshold cointegration models and the counterpart of the linear cointegration models. The results showed that no one particular threshold cointegration model dictated the overall forecasting accuracy. For different interest rates pairs under consideration, different threshold cointegration models offered a better fit. Moreover, all of the linear cointegration models performed relatively less accurate than the threshold cointegration models, which reinforce the empirical applications of the threshold cointegration models.

The strong evidence of threshold behavior is in contrast to previous work in credit dynamics. There is one immediate implication from this finding when considering NRM(2000)'s puzzle regarding how one could effectively model the time-varying correlation matrix. Some corporate bond pricing models, e.g., Das and Tufano (1996) and Jarrow, Lando, and Turnbull (1997), are based on the probability transition matrix to govern the dynamics of future debt ratings. However, both models employ an exogenously specified fixed correlation matrix between spreads and rates. In reality, it is not appropriate to fix the correlation matrix. NRM (2000) showed that the correlation between spreads and rates is time varying. NRM(2000) noted, "*it is unclear to how to parameterize the correlation between spreads and rates*." It might be that the threshold cointegration methodology is a powerful tool to model the time-varying correlation matrix between spreads and rates. One might estimate the historical data to obtain the model estimates and different correlation matrices under different regimes. Then, based on above information, combined with current rates and spreads, one might easily model the credit spread dynamics with time-varying correlation matrices.

Although convincing evidence has been presented in this research that threshold cointegration specification is a powerful tool for the empirical econometric application, there are still some possible future research issues. In the theoretic front, researchers might consider the following:

- When using Enders-Siklos model, extend the model to allow for two threshold variables so that the specification has a three-regime setting.
- When applying Hansen-Seo and Lo-Zivot specifications, allow for estimating cointegrating vector, delay variable and threshold variables simultaneously.
- For all threshold cointegration models in this study, extend the models to allow for at least three variables in the threshold cointegration framework, i.e., allowing for TVECMs with multiple cointegrating vectors.
- Develop a distribution theory for the parameter estimates for the threshold cointegration model.

In the empirical application side, researchers might consider the following:

- Extend the model setting to have multiple corporate bond indices to examine whether rates in one credit class contain information about the level and short run dynamics of rates in another credit class.
- Include some other macroeconomic variables in the threshold cointegration setting to control for economic evolution.
- Incorporate some other variables like, liquidity risk, default risk, the expected loss in the event of default into the specification to model the yield on risky debts.

APPENDIX A. ESTIMATION OF THE LO-ZIVOT MODEL

Based on sequential conditional least squares method, some of the previous research effort in the estimation of TAR and TVECM models include: Tong (1983) and Hansen (1999), for the estimation of univariate TAR models; Chan and Tsay (1998) and Berben and van Dijk (1998, 1999), for the estimation of continuous univariate TAR models. In Tsay (1998), the estimation of multivariate TVAR models is discussed. However, the estimation of a TVECM, with a known cointegrating vector, has not been formally discussed. Lo and Zivot (2001) combines Hansen's (1999) treatment of the estimation of TAR(2) and TAR(3) and Tsay's (1998) treatment of the estimation of multivariate TVAR models to estimate a TVECM model with one known cointegrating vector. A sketch of the Lo-Zivot estimation procedure is provided in this appendix.

Let x_t be a 2×1 I(1) time series, which is cointegrated with one known 2×1 cointegrating vector β. Let $w_t = \beta' x_t$ denote the I(0) error-correction term. Then, a TVECM (threshold vector error correction model) of order k at level can be written as:

$$\Delta x_t = \begin{cases} A_1' X_{t-1} + u_t, & \text{if} \quad w_{t-d} \le c, \\ A_2' X_{t-1} + u_t, & \text{if} \quad w_{t-d} > c, \end{cases} \tag{A.1}$$

where $X_{t-1}' = [1, w_{t-1}, \Delta x_{t-1}, \Delta x_{t-2}, ..., \Delta x_{t-k+1}]$, c is the threshold parameter, d is the delay parameter, an integer, and usually is assumed to be less than the order of autoregression in the model (k). With dimensions: X_{t-1} is $2k \times 1$, A_j is $2k \times 2$, where j = 1, 2.

One may rewrite above equation as:

$$\Delta x_t = A_1' X_{t-1} I_t(c,d) + A_2' X_{t-1}(1 - I_t(c,d)) + u_t, \tag{A.2}$$

where $I_t(c,d) = I(w_{t-d} \le c)$, and $I(\cdot)$ is the indicator function.

Lo and Zivot propose a two-step procedure to estimate model (A.2) by sequential multivariate least squares. The procedure states:

Step 1: Conditional on (c,d), the parameters (A_1, A_2) are estimated by multivariate least squares giving the residual sum of squares:

$$S_2(c,d) = \mathrm{trace}(\widehat{\Sigma}_2(c,d)),$$

where $\widehat{\Sigma}_2(c,d)$ is the multivariate least squares estimate of $\Sigma = \mathrm{var}(u_t)$, conditional on (c,d) for the two-regime TVECM.

Step 2: The least squares estimates of (c,d) are obtained as:

$$(\hat{c},\hat{d}) = \arg\min_{c,d} S_2(c,d),$$

by using a two-dimensional grid search to find the values of c and d. The final estimates of $\widehat{A}_j$ are given by $\widehat{A}_j = \widehat{A}_j(\hat{c},\hat{d})$, j = 1, 2, and the estimate of the residual covariance matrix is given by $\widehat{\Sigma}_2(\hat{c},\hat{d})$.

Next, consider a three-regime TVECM in the following:

$$\Delta x_t = \begin{cases} A_1' X_{t-1} + u_t, & \text{if} \quad w_{t-d} \le c^{(1)}, \\ A_2' X_{t-1} + u_t, & \text{if} \quad c^{(1)} < w_{t-d} \le c^{(2)}, \\ A_3' X_{t-1} + u_t, & \text{if} \quad w_{t-d} > c^{(2)}. \end{cases} \tag{A.3}$$

One can rewrite above model as:

$$\Delta x_t = A_1' X_{t-1} I_{1t}(c,d) + A_2' X_{t-1} I_{2t}(c,d) + A_3' X_{t-1} I_{3t}(c,d) + u_t,$$

where $c = (c^{(1)}, c^{(2)})$ and

$$I_{jt}(c^{(j-1)} < z_{t-d} \le c^{(j)}) = \begin{cases} 1, & \text{if } c^{(j-1)} < z_{t-d} \le c^{(j)}, j = 1,2,3, \\ 0, & \text{otherwise,} \end{cases}$$

where $-\infty = c^{(0)} < c^{(1)} < c^{(2)} < c^{(3)} = \infty$.

Similar two-step procedure may be used to estimate the parameters in above three-regime TVECM.

Step 1: Conditional on (c,d), estimate (A_1, A_2, A_3) by multivariate least squares, giving the residual sum of squares S_3(c,d);

$$S_3(c,d) = \text{trace}(\hat{\Sigma}_3(c,d)),$$

where $\hat{\Sigma}_3(c,d)$ is the multivariate least squares estimate of $\Sigma = \text{var}(u_t)$, conditional on (c,d) for the three-regime TVECM.

Step 2: Minimize S_3(c,d) by using a three-dimensional grid search to find the values of c and d.

Hansen (1999) recommends a two-step short cut to avoid the three-dimensional grid search. Since, if the grid points become large, the computation becomes more burdensome. Hansen's two-step strategy adopts the sequential estimation of multiple breakpoints method proposed by Bai (1997). Hansen's two-step procedure is:

Step 1: Estimate the mis-specified two-regime model to obtain least squares estimates $(\hat{c}_1, \hat{d})$.

Step 2: Estimate $c = (c^{(1)}, c^{(2)})$ by least squares on equation (A.3) imposing $d = \hat{d}$ and one element of c equals $\hat{c}_1$. The resulting estimate $\hat{c}_2$ will be consistent for the remaining element of the pair $(c^{(1)}, c^{(2)})$.

APPENDIX B. ESTIMATION OF THE HANSEN-SEO MODEL

Let x_t be a $p \times 1$ I(1) time series, which is cointegrated with one $p \times 1$ cointegrating vector β. Let $w_t(\beta) = \beta' x_t$ denote the I(0) error-correction term. Then, a linear VECM (vector error correction model) of order (L+1) can be written as:

$$\Delta x_t = A' X_{t-1}(\beta) + u_t, \qquad \text{(B.1)}$$

where $X_{t-1}'(\beta) = [1,\ w_{t-1}(\beta),\ \Delta x_{t-1},\ \Delta x_{t-2}, \ldots,\ \Delta x_{t-L}]$,

with dimensions: $X_{t-1}(\beta)$ is $k \times 1$, A is $k \times p$, and $k = pL+2$. The error term u_t is a $p \times 1$ Martingale difference sequence with finite variance matrix $\Sigma = E(u_t u_t')$ of dimension $p \times p$. Here, $w_{t-1}(\beta)$ and $X_{t-1}(\beta)$ indicate that the variables are evaluated at a generic value of β. When evaluated at the true value of the cointegrating vector, the variables are denoted as w_{t-1} and X_{t-1}, respectively.

To achieve identification, one needs to impose normalization restrictions on β. For the $p = 2$ case, since there is only one cointegrating vector, one element of β is set to equal unity. For $p > 2$, the restriction is imposed that in the i-th cointegrating vector, the coefficient on x_{it} enters with a nonzero coefficient, which one can normalize to 1, for $i = 1,\ldots,p$.

Under the assumption that the errors u_t are i.i.d. Gaussian, one can estimate the parameters (β, A, Σ) by MLE (maximum likelihood estimation). Let the estimates be denoted $(\tilde{\beta}, \tilde{A}, \tilde{\Sigma})$, and let $\tilde{u}_L = \Delta x_L - \tilde{A}' X_{L-1}(\tilde{\beta})$ be the residual vector.

A two-regime threshold cointegration model can be expressed as:

$$\Delta x_t = \begin{cases} A_1' X_{t-1}(\beta) + u_t, & \text{if} \quad w_{t-1}(\beta) \le \gamma, \\ A_2' X_{t-1}(\beta) + u_t, & \text{if} \quad w_{t-1}(\beta) > \gamma, \end{cases}$$

where γ is the threshold parameter. One may rewrite the above equation as:

$$\Delta x_t = A_1' X_{t-1}(\beta) d_{1t}(\beta,\gamma) + A_2' X_{t-1}(\beta) d_{2t}(\beta,\gamma) + u_t, \tag{B.2}$$

where $d_{1t}(\beta,\gamma) = I(w_{t-1}(\beta) \le \gamma)$, $d_{2t}(\beta,\gamma) = I(w_{t-1}(\beta) > \gamma)$, and $I(\cdot)$ is the indicator function. Notice that the value of the error correction term $w_{t-1}(\beta) = \beta' x_{t-1}$ defines the two regimes in the threshold model (B.2). The coefficient matrix A_1 is not necessarily equal to A_2.

The threshold effect has content, if $0 < \Pr(w_{t-1}(\beta) \le \gamma) < 1$, as Hansen and Seo indicate, otherwise the threshold cointegration model will collapse to a linear cointegration model. The following constraint was imposed to ensure the nonlinearity:

$$\pi_0 \le \Pr(w_{t-1} \le \gamma) \le 1 - \pi_0, \tag{B.3}$$

where $\pi_0 > 0$ is a trimming parameter. Notice that setting π_0 too close to zero will reduce testing power. Andrews (1993) suggests that setting π_0 between 0.05 and 0.15 is more appropriate.

Since u_t is i.i.d. Gaussian, the likelihood function is:

$$L_n(A_1, A_2, \Sigma, \beta, \gamma) = -\frac{n}{2}\log|\Sigma| - \frac{1}{2}\sum_{t=1}^{n} u_t(A_1, A_2, \beta, \gamma)' \Sigma^{-1} u_t(A_1, A_2, \beta, \gamma),$$

where: $u_t(A_1, A_2, \beta, \gamma) = \Delta x_t - A_1' X_{t-1}(\beta) d_{1t}(\beta,\gamma) - A_2' X_{t-1}(\beta) d_{2t}(\beta,\gamma)$. The MLE $(\hat{A}_1, \hat{A}_2, \hat{\Sigma}, \hat{\beta}, \hat{\gamma})$ are the values that maximize $L_n(A_1, A_2, \Sigma, \beta, \gamma)$.

Estimation procedure

Step 1: First, concentrate out (A_1, A_2, Σ) by holding (β, γ) fixed and compute the constrained MLE for (A_1, A_2, Σ).

Through appropriate constrained OLS estimation, one obtains:

$$\hat{A}_1(\beta,\gamma)=\left(\sum_{t=1}^{n}X_{t-1}(\beta)X_{t-1}(\beta)'d_{1t}(\beta,\gamma)\right)^{-1}\left(\sum_{t=1}^{n}X_{t-1}(\beta)\Delta x_t d_{1t}(\beta,\gamma)\right), \tag{B.4}$$

$$\hat{A}_2(\beta,\gamma)=\left(\sum_{t=1}^{n}X_{t-1}(\beta)X_{t-1}(\beta)'d_{2t}(\beta,\gamma)\right)^{-1}\left(\sum_{t=1}^{n}X_{t-1}(\beta)\Delta x_t d_{2t}(\beta,\gamma)\right), \tag{B.5}$$

$$\hat{u}_t(\beta,\gamma)=u_t(\hat{A}_1(\beta,\gamma),\ \hat{A}_2(\beta,\gamma),\beta,\gamma),$$

and

$$\hat{\Sigma}(\beta,\gamma)=\frac{1}{n}\sum_{t=1}^{n}\hat{u}_t(\beta,\gamma)\hat{u}_t(\beta,\gamma)'. \tag{B.6}$$

Notice that (B.4) (or (B.5)) is the OLS regressions of Δx_t on $X_{t-1}(\beta)$ for the subsample for which $w_{t-1}(\beta)\leq\gamma$ (or $w_{t-1}(\beta)>\gamma$).

The concentrated likelihood function is:

$$L_n(\beta,\gamma)=L_n(\hat{A}_1(\beta,\gamma),\ \hat{A}_2(\beta,\gamma),\ \hat{\Sigma}(\beta,\gamma),\ \beta,\ \gamma)=-\frac{n}{2}\log\left|\hat{\Sigma}(\beta,\gamma)\right|-\frac{np}{2}. \tag{B.7}$$

Step 2: Compute the vector of parameters: $(\hat{\beta},\hat{\gamma})$.

The MLE $(\hat{\beta},\hat{\gamma})$ are the minimizers of $\log\left|\hat{\Sigma}(\beta,\gamma)\right|$ subject to the normalization imposed on β and the constraint: $\pi_0\leq n^{-1}\sum_{t=1}^{n}I(\beta'x_t\leq\gamma)\leq 1-\pi_0$. The MLE for A_1 and A_2 are thus $\hat{A}_1=\hat{A}_1(\hat{\beta},\hat{\gamma})$, and $\hat{A}_2=\hat{A}_2(\hat{\beta},\hat{\gamma})$.

Since the concentrated likelihood function is not smooth, in the case of $p = 2$, Hansen and Seo suggest using a grid search over the two-dimensional space (β, γ). For higher dimension cases ($p > 2$), they suggest a generic algorithm might be appropriate.

Hansen-Seo algorithm for the p = 2 case

1. Use OLS applied to the linear cointegration model to obtain a consistent initial estimate of β, $\tilde{\beta}$.
2. Construct a larger confidence interval $[\beta_L, \beta_U]$ for β constructed from $\tilde{\beta}$. Two end points of the larger confidence interval are: $\beta_L = \tilde{\beta} - 6*se(\tilde{\beta})$ and $\beta_U = \tilde{\beta} + 6*se(\tilde{\beta})$, where $se(\tilde{\beta})$ is the estimated standard error of $\tilde{\beta}$ from step 1.
3. Form an evenly spaced grid on $[\beta_L, \beta_U]$.
4. Compute $\tilde{w}_{t-1} \equiv w_{t-1}(\tilde{\beta}) = \tilde{\beta}'x_{t-1}$ and order the series from low to high (or high to low).
5. The search region $[\gamma_L, \gamma_U]$ is set so that γ_L is the π_0 percentile of $\tilde{w}_{t-1}$, and γ_U is the $(1 - \pi_0)$ percentile. This step imposes the constraint (B.3): $\pi_0 \le \Pr(w_{t-1} \le \gamma) \le 1 - \pi_0$.
6. Form an evenly spaced grid on $[\gamma_L, \gamma_U]$.
7. Form a two-dimensional grid on $[\beta_L, \beta_U] \times [\gamma_L, \gamma_U]$.
8. For each pair of (β, γ) on the grid, compute $\hat{A}_1(\beta,\gamma)$, $\hat{A}_2(\beta,\gamma)$, and $\hat{\Sigma}(\beta,\gamma)$ as defined in equations (B.4), (B.5), and (B.6).
9. Find $(\hat{\beta}, \hat{\gamma})$ as the values of (β,γ) on this grid that yields the lowest value of $\log\left|\hat{\Sigma}(\beta,\gamma)\right|$.
10. Compute $\hat{\Sigma} = \hat{\Sigma}(\hat{\beta},\hat{\gamma})$, $\hat{A}_1 = \hat{A}_1(\hat{\beta},\hat{\gamma})$, $\hat{A}_2 = \hat{A}_2(\hat{\beta},\hat{\gamma})$, and $\hat{u}_t = \hat{u}_t(\hat{\beta},\hat{\gamma})$.
11. Refine the grid search and repeat step 1 to step 10 to see if the solution is sensitive to the particular grid chosen.

APPENDIX C. SOME MEASURES OF FORECAST ACCURACY

Let y_t be the covariance stationary time series and let $y_{t+h,t}$ be the h-step-ahead linear squares forecast. Then, the corresponding h-step-ahead forecast error is: $e_{t+h,t} = y_{t+h} - y_{t+h,t}$. Define the corresponding h-step-ahead forecast percent error as: $p_{t+h,t} = (y_{t+h} - y_{t+h,t})/y_{t+h}$. One may define the next few accuracy measures as:

Mean error: $ME = \frac{1}{T}\sum_{t=1}^{T} e_{t+h,t}$, measures bias. Other things being equal, a forecast with a small bias is preferred.

Mean percent error: $MPE = \frac{1}{T}\sum_{t=1}^{T} p_{t+h,t}$, measures bias. Other things being equal, a forecast with a small bias is preferred.

Error variance: $EV = \frac{1}{T}\sum_{t=1}^{T} (e_{t+h,t} - ME)^2$, measures dispersion of the forecast errors. Other things being equal, a forecast is preferable whose errors have small variance.

However, ME and EV are components of accuracy, but neither provides an overall accuracy measure. The most common overall accuracy measures are MSE and MSPE:

Mean squared error: $MSE = \frac{1}{T}\sum_{t=1}^{T} e_{t+h,t}^2$.

Mean squared percent error: $MPSE = \frac{1}{T}\sum_{t=1}^{T} p_{t+h,t}^2$.

Sometimes, the square roots of MSE and MPSE are used to preserve units, yielding the RMSE and RMSPE:

Root mean squared error: $RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T} e_{t+h,t}^2}$.

Root mean squared percent error: $\text{RMSPE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T} p_{t+h,t}^2}$.

Other common accuracy measures are MAE and MAPE:

Mean absolute error: $\text{MAE} = \frac{1}{T}\sum_{t=1}^{T} \left|e_{t+h,t}\right|$.

Mean absolute percent error: $\text{MAPE} = \frac{1}{T}\sum_{t=1}^{T} \left|p_{t+h,t}\right|$.

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